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THE

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COMMERCIAL ARITHMETIC

(ADVANCE SHEETS.)

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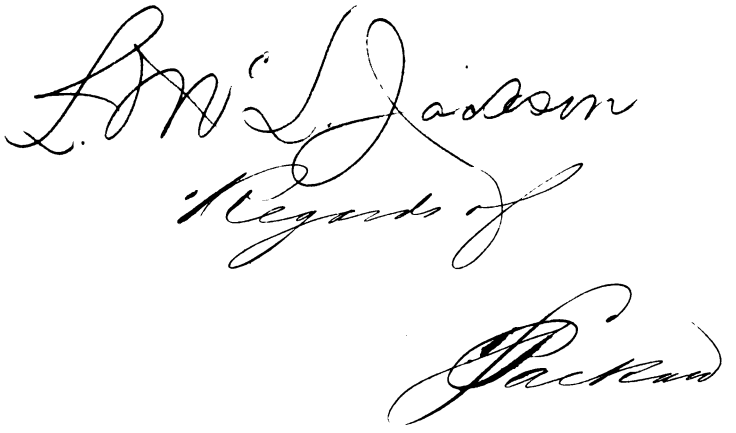

THE

PACKARD

COMMERCIAL ARITHMETIC.

BY

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[ADVANCE SHEETS.]

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EXPLANATORY.

THE 200 pages of arithmetical matter here printed comprise two-thirds of a book now in preparation, and to be completed about April 1, 1882. The authors of this new Commercial Arithmetic—as it will be—have not stopped to inquire whether the educational world is standing still for want of such a work, or whether the embarrassment lies in the direction of a surfeit; and although not indifferent to these considerations, they have had nothing to do with the inception or growth of the book. It is not intended to supply any general want, but from the beginning to the end will be adapted to a known and special want—that of the mathematical department of Packard's Business College; and it is but just to say that had this want been met by any book already published the present work would not have been undertaken.

This is not the place nor the time to speak of the characteristics of the book; and in fact, if they do not speak for themselves no advertisement by the authors can supply the deficiency. It is proper to say, however, that it is not and will not be a compilation from other published works; that the facts which it presents concerning business laws and business customs have all been ascertained by personal inquiry and verified by official correspondence; and that the examples contain no puzzles nor conundrums, but are all practical and reasonable, and for the most part gathered from active business. Whether they are fuller and better than those of other books, or whether the subjects are presented in the proper order and with the proper balance and symmetry, are professional questions which every one can answer for himself.

If any teachers to whom these advance sheets are sent see in them the promise of what they may possibly need, there will be no objection to their saying so, and no objection on the part of the authors to their using the completed book when it is ready.

The supplementary table of contents indicates the character and scope of the remaining hundred pages.

S. S. PACKARD,
BYRON HORTON.

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GENERAL AVERAGE.
CLEARING HOUSES.
VARIOUS SHORT METHODS.

COMMERCIAL ARITHMETIC.

PROPERTIES OF NUMBERS.

Definitions.

1. A **Unit**, or **Unity**, is one, or a single thing; as *one*, *one* foot, *one* dollar.

2. A **Number** is a unit, or a collection of units; as *one*, *four*, *three* feet, *five* dollars.

3. All numbers are either *integral* or *fractional*, *abstract* or *concrete*.

4. An **Integral Number**, or **Integer** is a number which expresses whole things; as *two*, *four* gallons, *seven* dollars.

5. A **Fractional Number**, or **Fraction** is a number which expresses one or more equal parts of a unit; as *one-half*, *three-fourths*.

6. An **Abstract Number** is a number which does not refer to any particular object; as *one*, *six*, *ten*.

7. A **Concrete Number** is a number applied to an object, or quantity; as *three* apples, *five* pounds, *ten* dollars.

8. Integral numbers are either *odd* or *even*, *prime* or *composite*.

9. An **Odd Number** is a number whose unit figure is 1, 3, 5, 7, or 9; as 7, 21, 39.

10. An **Even Number** is a number whose unit figure is 0, 2, 4, 6, or 8; as 6, 40, 74.

11. A **Prime Number** is a number which can be exactly divided only by itself and unity; as 1, 7, 13, 29.

12. Numbers are **prime to each other** when no integral number greater than 1 will divide each without a remainder.

Numbers that are prime to each other are not necessarily prime numbers. Thus, 25 and 28 are prime to each other, but they are not prime numbers.

13. A **Composite Number** is a number which can be exactly divided by other integers besides itself and unity.

Thus 28, the product of 4 and 7, is a composite number. It is exactly divisible by 4 and 7.

Divisibility of Numbers.

14. An **Exact Divisor** of a number is any number that will divide it without a remainder.

Thus 2, 3, 4, 6, 8, and 12 are exact divisors of 24.

15. A number is said to be *divisible* by another when the latter will divide the former without a remainder. Any number is divisible

1. By 2, if it is an even number; as 6, 28, and 32.

2. By 3, if the sum of its digits is divisible by 3; as 849 ($8+4+9=21$, 21 is divisible by 3), 7323, and 47892.

3. By 4, if the two right-hand figures are ciphers, or express a number divisible by 4; as 1100, 216, and 7328.

4. By 5, if the right-hand figure is 0 or 5; as 40 and 135.

5. By 6, if it is an even number and the sum of its digits is divisible by 3; as 216, 840, and 732.

6. By 8, if the three right-hand figures are ciphers, or express a number divisible by 8; as 3000 and 7168.

7. By 9, if the sum of its digits is divisible by 9; as 216, 783, and 12348.

Prime Factors.

16. The **Factors** of a number are those numbers which multiplied together will produce the number.

Thus 4 and 7; 2 and 14; 2, 2, and 7 are factors of 28. The number itself and unity are not regarded as factors.

The *factors* of a number are also the *exact divisors* of it.

17. A **Prime Factor** is a prime number used as a factor.

Thus, 2, 2, and 7 are the prime factors of 28. 4 is a factor of 28, but not a *prime* factor.

18. To find all the prime factors of a composite number.

Ex. What are the prime factors of 6930.

OPERATION.

2) 6930
3) 3465
3) 1155
5) 385
7) 77
11

ANALYSIS.—Any prime number that is an exact divisor of the given number is a prime factor of it. Divide the given number by 2 (**15**, 1), the least prime divisor of it, obtaining the quotient 3465. Next, divide this quotient successively by 3 (**15**, 2), 3, 5 (**15**, 4), and 7. The last quotient 11 is a prime number and therefore a prime factor. The several divisors 2, 3, 3, 5, 7 and the last quotient 11 are the prime factors required.

$2 \times 3 \times 3 \times 5 \times 7 \times 11 = 6930.$

19. RULE.—*Divide by the least prime number which will divide the given number without a remainder. In like manner divide the resulting quotient, and continue the division until the quotient is a prime number. The several divisors and the last quotient are the prime factors.*

EXAMPLES.

20. Resolve the following numbers into their prime factors:

1.	3465.	*7.*	6552.	*13.*	8192.	*19.*	6660.
2.	3003.	*8.*	7826.	*14.*	6561.	*20.*	2448.
3.	4158.	*9.*	6006.	*15.*	3125.	*21.*	8525.
4.	3150.	*10.*	5368.	*16.*	1800.	*22.*	9936.
5.	3675.	*11.*	3825.	*17.*	1935.	*23.*	9576.
6.	2310.	*12.*	5324.	*18.*	2475.	*24.*	5075.

Common Divisors.

21. A **Common Divisor**, or **Common Measure**, of two or more numbers is any number that will divide each without a remainder; hence it is a common factor of each of them.

22. The **Greatest Common Divisor** of two or more numbers is the *greatest* number that will divide each without a remainder; hence it is their greatest common factor.

Thus, 2, 3, 4, and 12 are common divisors of 36, 48, and 60; 12 is their greatest common divisor.

23. Principle.—*The greatest common divisor of two or more numbers is the product of all their common prime factors.*

24. To find the greatest common divisor of two or more numbers.

Ex. What is the greatest common divisor of 168, 252, and 420?

FIRST OPERATION.

$168 = \overset{*}{2} \times \overset{*}{2} \times 2 \times \overset{*}{3} \times \overset{*}{7}$

$252 = \overset{*}{2} \times \overset{*}{2} \times \overset{*}{3} \times 3 \times \overset{*}{7}$

$420 = \overset{*}{2} \times \overset{*}{2} \times \overset{*}{3} \times 5 \times \overset{*}{7}$

$2 \times 2 \times 3 \times 7 = 84.$

Analysis.—Resolve the numbers into their prime factors. The product, 84, of the common factors 2, 2, 3, and 7 is the greatest common divisor. (*Prin.*)

SECOND OPERATION.

4) 168, 252, 420

7) 42, 63, 105

3) 6, 9, 15

2, 3, 5

$4 \times 7 \times 3 = 84.$

Analysis.—Divide the given numbers by any number that will divide them all without a remainder, and divide the quotients in the same manner until the last quotients have no common divisor. Since 4 will divide all the given numbers, and 3 and 7 will divide successively the resulting quotients, their product, 84, is a common divisor of the given numbers. Since the last quotients have no common divisor or factor, 84 is the *greatest* common divisor.

25. Rule.—*Resolve the numbers into their prime factors. The product of the factors common to all the numbers will be the greatest common divisor.* Or,

Divide the given numbers by any factor that will divide all of them without a remainder. In like manner divide

the resulting quotients, and continue the division until the quotients have no common factor. The product of the several divisors will be the greatest common divisor.

EXAMPLES.

26. Find the greatest common divisor of the following numbers:

1.	24, 36, and 48.	*9.*	108, 144, and 360.
2.	35, 56, and 70.	*10.*	144, 336, and 240.
3.	42, 56, and 28.	*11.*	165, 550, and 220.
4.	30, 60, and 75.	*12.*	792, 144, and 216.
5.	64, 96, and 128.	*13.*	405, 243, and 324.
6.	66, 198, and 330.	*14.*	378, 126, and 252.
7.	90, 150, and 210.	*15.*	375, 625, and 250.
8.	84, 420, and 126.	*16.*	288, 720, and 864.

27. To find the greatest common divisor of two numbers when they are not readily factored.

28. PRINCIPLES.—1. *If the smaller of two numbers is a divisor of the greater, it is their greatest common divisor.*

2. *A common divisor of two numbers is a divisor of their sum, and also of their difference.*

3. *A divisor of a number is a divisor of any multiple of that number.*

29. RULE.—*Divide the greater number by the smaller, and divide the last divisor by the remainder; and so continue until there is no remainder. The last divisor will be the greatest common divisor.*

NOTES.—1. When the greatest common divisor of more than two numbers is required, find the greatest common divisor of the smallest two first, and of this greatest common divisor and the next greater, and so on, until all the numbers are used. The last divisor will be the greatest common divisor of all the given numbers.

2. If, at any step in the process, a prime factor appear that is not common to all the numbers, it may be rejected. (See second operation of example.)

3. If the remainder at any time is a prime number, and it is not contained in the last divisor, there is no common divisor greater than 1; it will therefore be useless to further continue the division.

Ex. Find the greatest common divisor of 391 and 437.

OPERATIONS.

391) 437 (1
391
46) 391 (8
368
23) 46 (2
46
0

Or,

2) 46
23) 391 (17
23
161
161
0

DEMONSTRATION. — Since 23 is a divisor of 46, it is a divisor of 368, a multiple of 46 (*Prin.* 3). Since 23 is a divisor of itself and 368, it is a divisor of their sum, 391 (*Prin.* 2). Since 23 is a divisor of 46 and 391, it is a divisor of their sum, 437. 23 is therefore a *common divisor* of 391 and 437, the given numbers.

The greatest common divisor of 391 and 437, whatever it may be, is a divisor of their difference, 46 (*Prin.* 2); also of 368, a multiple of 46 (*Prin.* 3); also of 23, 391 − 368 (*Prin.* 2). Since the divisor of a number cannot be greater than itself, the greatest common divisor of the given numbers cannot be greater than 23. 23 is therefore the *greatest common divisor*.

30. Find the greatest common divisor of the following numbers:

1. 319 and 377.
2. 259 and 629.
3. 589 and 713.
4. 903 and 989.
5. 611, 799, and 987.
6. 744, 984, and 522.
7. 391, 667, and 920.
8. 451, 481, and 737.
9. 504, 756, and 252.
10. 425, 748, and 561.

COMMON MULTIPLES.

31. A **Multiple** of a number is a number that is exactly divisible by it; or, it is any product of which the given number is a factor.

32. A **Common Multiple** of two or more numbers is a number that is exactly divisible by each of them.

33. The **Least Common Multiple** of two or more numbers is the *least* number that is exactly divisible by each of them.

Thus, 12, 24, 36, and 48 are common multiples of 4 and 6; 12 is their least common multiple.

34. PRINCIPLES.—1. *A multiple of a number contains all the prime factors of that number.*

2. *A common multiple of two or more numbers contains all the prime factors of each of those numbers.*

3. *The least common multiple of two or more numbers contains all the prime factors of each of the numbers, and no other factors.*

35. To find the least common multiple of two or more numbers.

Ex. What is the least common multiple of 12, 18, 20, and 40?

FIRST OPERATION.

$12 = 2 \times 2 \times 3$
$18 = 2 \times 3 \times 3$
$40 = 2 \times 2 \times 2 \times 5$
$2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$

ANALYSIS.—Since 40, a multiple of 20, contains all the prime factors of 20, the number 20 may be omitted in the operation. Resolve the numbers into their prime factors. The least common multiple must contain 2 as a factor 3 times in order to be divisible by 40; it must contain 3 as a factor twice in order to be divisible by 18; and it must contain 5 as a factor, in order to be divisible by 40. 360, the product of the factors 2, 2, 2, 3, 3, and 5, is the least common multiple of the given numbers, since it contains the different factors the greatest number of times that they occur in the given numbers, and no other factors (*Prin.* 3).

SECOND OPERATION.

$2 \mid 12,\ 18,\ 40$
$2 \mid 6,\ 9,\ 20$
$3 \mid 3,\ 9,\ 10$
$1,\ 3,\ 10$
$2 \times 2 \times 3 \times 3 \times 10 = 360$

ANALYSIS.—The factors of the required multiple may be selected by the following process. Divide the given numbers by any prime number that will divide two or more of them, writing the quotients and the undivided numbers beneath. Treat the resulting numbers in like manner, and continue the process until no two of the numbers have a common factor or divisor. The product of the several divisors and the remaining quotients and undivided numbers will be the least common multiple.

36. RULE.—*Resolve the given numbers into their prime factors. The product of the different prime factors, taking each factor the greatest number of times it appears in any of the numbers, will be the least common multiple.* Or,

Divide the given numbers by any prime number (see Note 2) *that will exactly divide two or more of them, writing the quotients and undivided numbers beneath. Repeat the operation with the resulting numbers until there is no exact divisor of any two of them. The product of the several divisors and the last quotients and undivided numbers will be the least common multiple.*

NOTES.—1. In the operation, reject such of the smaller numbers as are divisors of the larger; also reject such of the quotients and undivided numbers as are divisors of the others.

2. Divide by composite numbers when they are exact divisors of all the numbers.

EXAMPLES.

37. Find the least common multiple of the following numbers:

1.	2, 3, 4, 5, and 6.	*15.*	18, 24, and 36.
2.	8, 10, 12, and 15.	*16.*	10, 24, and 32.
3.	12, 15, 18, and 24.	*17.*	16, 18, and 20.
4.	6, 10, 15, and 30.	*18.*	24, 36, and 40.
5.	16, 24, and 48.	*19.*	32, 48, and 72.
6.	30, 40, and 60.	*20.*	16, 22, and 24.
7.	2, 4, 8, and 16.	*21.*	18, 28, and 30.
8.	14, 21, and 28.	*22.*	12, 16, and 20.
9.	5, 8, 15, and 18.	*23.*	33, 44, and 55.
10.	6, 9, 21, and 24.	*24.*	27, 36, and 42.
11.	12, 20, and 30.	*25.*	36, 45, and 60.
12.	6, 10, 30, and 40.	*26.*	28, 35, and 42.
13.	32, 48, and 60.	*27.*	45, 55, and 60.
14.	24, 32, and 40.	*28.*	60, 72, and 84.

CANCELLATION.

38. **Cancellation** is a method of shortening an operation by rejecting equal factors from both dividend and divisor.

39. PRINCIPLES.—1. *Canceling or rejecting a factor from a number, divides the number by that factor.*

2. *Dividing both dividend and divisor by the same number does not affect the value of the quotient.*

Ex. Divide 84×36 by 27×14.

OPERATIONS.

$$\frac{\overset{\overset{2}{\cancel{6}}}{\cancel{84}} \times \overset{4}{\cancel{36}}}{\underset{\cancel{3}}{\cancel{27}} \times \cancel{14}} = 8$$

Or,

$$\begin{array}{r|l} {}^{3}\cancel{27} & \cancel{84}\,{}^{\cancel{6}\,2} \\ \cancel{14} & \cancel{36}\,{}^{4} \\ \hline & 8 \end{array}$$

ANALYSIS.—Indicate the operations to be performed as in the margin. It is seen by inspection that 36 and 27 contain the common factor 9; therefore cancel or reject it from both, retaining the factors 4 and 3 respectively. 14 and 84 contain the common factor 14; therefore reject it, retaining the factor 6 in the dividend. [Since cancellation is a process of division, the rejecting of 14 does not destroy it, but divides it, leaving 1 as a quotient. It is unnecessary to write 1 as a quotient, except when there are no other factors in the dividend.] 3 is a common factor of 6 and 3; therefore reject it from both, retaining the factor 2 in the dividend. The product of the remaining factors, 2 and 4, is the required quotient.

40. RULE.—*Indicate the operations to be performed by writing the numbers denoting multiplication above a horizontal line, and the numbers denoting division below. The numbers above the line will form a dividend, and the numbers below a divisor. Cancel or reject the factors common to both dividend and divisor. The product of the remaining factors of the dividend divided by the product of the remaining factors of the divisor will be the required quotient.*

EXAMPLES.

41. *1.* Divide $27 \times 48 \times 60$ by $54 \times 36 \times 40$.

What is the value of the following expressions:

2. $\dfrac{40 \times 36 \times 42 \times 18}{9 \times 35 \times 30 \times 8}$.

3. $\dfrac{24 \times 30 \times 54 \times 35}{14 \times 15 \times 21 \times 64}$.

4. $\dfrac{360 \times 28 \times 27 \times 5}{25 \times 42 \times 18 \times 12}$.

5. $\dfrac{17 \times 36 \times 25 \times 144}{48 \times 60 \times 108 \times 51}$.

6. $\dfrac{1760 \times 175 \times 6}{4 \times 9 \times 100 \times 10}$.

7. $\dfrac{1760 \times 6 \times 145}{100 \times 365}$.

8. $\dfrac{1500 \times 144 \times 5}{365 \times 100}$.

9. $\dfrac{144 \times 625 \times 37 \times 12}{288 \times 375 \times 185}$.

10. Multiply 72 by 3×18, divide the product by 8 times 9, multiply the quotient by 7×20, divide the product by 360, multiply the quotient by 6 times 8.

11. If 42 tons of coal cost $147, what will 16 tons cost?

12. A man gave 9 pounds of butter at 17 cents a pound for 3 gallons of molasses; how much was the molasses worth a gallon?

13. If 20 pounds of beef cost 250 cents, what cost 75 pounds?

14. How many potatoes at 65 cents per bushel will pay for 13 weeks' board at $7.50 per week?

15. A merchant bought 375 barrels of flour at $5.50 per barrel, and paid in cloth at $2.75 per yard; how many yards did it require?

16. How many pounds of coffee at 27 cents per pound should be given for 57 bushels of corn at 63 cents per bushel?

17. Sold 28 bushels of apples for $21; what should I receive for 42 bushels?

18. How many cows worth $35 each must be given in exchange for 84 tons of hay at $15 per ton?

19. How many bushels of corn at 52 cents a bushel must be exchanged for 324 bushels of oats at 39 cents per bushel?

20. If 430 bushels of wheat are obtained from sowing 7 bushels, how much would be obtained from sowing 21 bushels?

21. What should be paid for the transportation of 3600 pounds of cheese at the rate of 47 cents per 100 pounds?

22. What must be paid for transporting 31600 pounds of iron at $5 per ton of 2000 pounds?

23. What will 7840 pounds of coal cost, at $6 per ton of 2240 pounds?

24. If 3 men eat 7 pounds of meat in one week, how much would 6 men eat in 4 weeks?

25. How many canisters, each holding 40 ounces, can be filled from 3 chests of tea, each containing 55 pounds of 16 ounces?

26. How many times can 16 bottles, each holding 3 pints, be filled from 6 demijohns, each containing 10 gallons of 8 pints each?

27. A man exchanged 275 barrels of potatoes, each containing 3 bushels, at 54 cents per bushel, for a certain number of pieces of muslin each containing 45 yards, at 11 cents per yard. How many yards did he receive?

28. If a person travel 24 hours each day at the rate of 45 miles an hour, how many days would it require to pass around the globe, a distance of 25000 miles?

REVIEW EXAMPLES.

42. *1.* Write in figures each of the following numbers, add them, and express in words (or numerate) their sum: Forty-five thousand and forty-five; sixteen thousand three hundred and sixty; one hundred and sixty-seven thousand; eight hundred and fifty thousand and ninety-two; nine million and twenty-four.

2. Subtract eight hundred and fourteen thousand nine hundred and sixteen from four million and nineteen thousand.

3. Multiply five hundred and sixty thousand seven hundred and eight by eighteen hundred and sixty.

4. A quantity of merchandise was bought for $27618.75, and sold for $32418.25. What was the gain?

5. What is the sum of 2817, 273, 30006, 97, 7285, 2700576, 7000781, 27?

6. If I sell goods for $23876, and gain $5389, what did the goods cost me?

7. What is the sum of the prime numbers from 20 to 50?

Add the following numbers as they stand, from left to right, and from right to left. [In making out bills, and in other commercial operations, a great deal of time can be saved by adding in this manner, without re-arranging the numbers.]

8. 17, 27, 36, 14, 43, 42, 65, 73, 81, 35.

9. 137, 414, 528, 345, 678, 975, 864, 357, 121, 234.

10. 67.16, 5.12, 3.75, 475, 38.42, 59.27, 38.75, 175.25.

11. 2345, 16, 375, 4218, 376, 7, 8475, 247, 39.

12. 1234.27, 348.25, 775, 7.16, 89.76, 374.12, 5673.56, 397.23.

Find the difference between the numbers in each of the following groups. [In all of these cases the subtrahend is placed above the minuend, the purpose being to give the student practice in subtracting *downward* rather than upward, as the general custom is. It is often requisite in business to perform the work in this way, and the accountant should practice both methods.]

(*13.*)	(*14.*)	(*15.*)	(*16.*)	(*17.*)
76534	19827	26347	72016	12345
81279	84362	71356	99385	54321

18. One factor of a certain number is 217 and the other 5280; what is the number?

19. Find the prime factors of 108108.

20. If the quotient is 375 and the divisor 246, what is the dividend?

21. If the product of two factors is 450072, and one of the factors is 987, what is the other factor?

22. What is the sum of the composite numbers from 60 to 90 inclusive?

23. Divide 76432801 by 783. Prove that your solution is correct.

24. A clerk receiving a salary of $1256, pays $468 a year for board, $180 for clothing, and $150 for other expenses. What amount has he left?

25. What is the least number that can be exactly divided by each of the following numbers: 24, 32, 80, 48, and 90?

26. If I take 24889 from the sum of 9872 and 24967, divide the remainder by 50, and multiply the quotient by 18, what is the product?

27. If 160 acres of land cost $10720, how many acres can be bought for $8844?

28. What is the least common multiple of the nine digits?

29. If 75 head of cattle cost $2550, what will 59 head cost?

30. A merchant sold 426 barrels of flour for $2556, which was $639 more than he gave for it. What did it cost him a barrel?

31. What is the greatest number that will exactly divide each of the following numbers: 246, 744, and 522?

32. What is the smallest sum of money with which horses can be bought at $96 each, cows at $30 each, or sheep at $5 each, using the same amount in each case?

33. A merchant bought 387 yards of cloth at 79 cts. per yard; he sold 298 yards at $1.16 per yard, and the remainder at 97 cts. per yard; how much did he gain?

34. Cash on hand at beginning of the day, $6492.75; cash received, $11456.75; cash paid out, $13285.26. Required the cash balance at the end of the day.

35. Mr. A has three farms, the first of which contains 158 acres, the second 32 acres less than the first, and the third as many as the other two. What is the value per acre, if all are worth $26128?

36. There are five bidders to supply the government with 800 tons Lehigh, 500 tons Cumberland, and 700 tons Baltimore coal. A offers Lehigh at $6.29, Cumberland at $4.38, and Baltimore at $7.23. B offers Lehigh at $6.80, Cumberland at $4.12, and Baltimore at $7.24. C offers Lehigh at $6.40, Cumberland at $4.45, and Baltimore at $7.18. D offers Lehigh at $6.17, Cumberland at $4.19, Baltimore at $7.20. E offers Lehigh at $6.50, Cumberland at $4.33, and Baltimore at $7.25. Who is the lowest bidder for the whole amount, and how much does each bid amount to?

37. A drover bought a number of cattle for $12204, and sold the same for $13560, by which he gained $4 per head. How many cattle were purchased?

38. A farmer raised in one year 512 bushels of wheat, the next year twice as much as he raised the first year, and the third year four times as much as he did the second year. What was the value of the three crops at $1.65 per bushel?

39. How many pounds of tea at 78 cts. per pound must be given for 375 bushels of wheat at $1.56 per bushel?

40. Bought 75 tons of hay at $16 per ton; gave in payment 56 sheep at $3.75 each, and the remainder I paid in butter at 33 cts. per pound. How many pounds of butter were required?

41. Bought 225 acres of land for $12600, and sold 116 acres at $65 per acre, and the remainder at cost; how much did I gain?

42. The estimated number of bushels of corn produced in the United States in 1877 was 1,342,558,000, the total value of crop was $480,643,400, and the total area of crop was 50,369,113 acres. What was the average value per bushel, and average value of yield per acre?

43. In 1878 there were 39258 postmasters in the United States, and their total salaries were $7,977,852; what was the average salary paid?

44. July 1, 1866, the public debt of the United States was $2,773,236,173, and May 1, 1880, $1,968,314,753; what was the average monthly decrease?

45. A sold to B 175 acres of land at $135 an acre, and by so doing gained $1925; B sold the land at a loss of $1750. What did A pay per acre, and what was B's selling-price per acre?

46. A merchant sold 800 barrels of flour for $5867, 144 barrels of which he sold at $7 per barrel, and 225 barrels at $6.75. At how much per barrel did he sell the remainder?

FRACTIONS.

DEFINITIONS.

43. A **Fraction** is one or more of the equal parts of a unit; as *one-half* ($\frac{1}{2}$), *two-thirds* ($\frac{2}{3}$), *one-fourth* ($\frac{1}{4}$), *seven-eighths* ($\frac{7}{8}$).

If a unit be divided into four equal parts, each part is called a fourth. If one of these parts be taken, the expression will be one-fourth ($\frac{1}{4}$); if three parts, three-fourths ($\frac{3}{4}$), etc.

44. The greater the number of equal parts into which a unit is divided, the less will be each part; the less the number of parts, the greater will be each part.

One-half ($\frac{1}{2}$) is greater than one-third ($\frac{1}{3}$); one-fourth ($\frac{1}{4}$) is less than one-third ($\frac{1}{3}$).

45. A fraction is usually expressed by two numbers, one written above the other, with a line between. Fractions written in this form are usually called **Common Fractions.**

46. The number below the line is called the **Denominator,** because while indicating the number of equal parts into which the unit is divided, it *denominates* or names those parts.

47. The number above the line is called the **Numerator,** because it shows how many of the parts are taken to form the fraction.

48. The numerator and denominator, taken together, are called the **Terms** of the fraction.

In the fraction three-fourths ($\frac{3}{4}$), 3 and 4 are the terms; 4 is the denominator, and shows that the unit is divided into four equal parts, called fourths; 3 is the numerator, and shows that three of these parts are taken to constitute the fraction.

49. A fraction is an expression of unperformed division. The numerator is the dividend, the denominator is the divisor, and the value of the fraction is the quotient.

50. A **Simple Fraction** is a single fraction, both of whose terms are integers.

51. Simple fractions are *proper* or *improper.*

52. A **Proper Fraction** is one that is less than a unit; the numerator being less than the denominator. Thus, $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{7}{8}$ are proper fractions.

53. An **Improper Fraction** is one that is equal to, or greater than a unit; hence the numerator must be equal to, or greater than the denominator. Thus, $\frac{3}{2}$, $\frac{5}{4}$, $\frac{8}{8}$, and $\frac{11}{8}$ are improper fractions.

54. A **Mixed Number** is an integer and a fraction united; as $2\frac{1}{2}$, $4\frac{3}{4}$, $18\frac{7}{8}$.

55. A **Compound Fraction** is a fraction of a fraction; as $\frac{1}{2}$ of $\frac{3}{8}$, $\frac{4}{5}$ of $7\frac{1}{2}$, $\frac{2}{3}$ of $\frac{5}{6}$.

56. A **Complex Fraction** is one whose numerator is a fraction or mixed number; as $\frac{\frac{3}{4}}{3}$, $\frac{105\frac{3}{5}}{12}$, $\frac{75\frac{3}{4}}{16}$, $\frac{3\frac{2}{3}}{5}$, $\frac{12\frac{1}{2}}{15}$.

The expression $\frac{3\frac{3}{4}}{5\frac{1}{2}}$ indicates division, and is not properly a fraction. A unit cannot be divided into $5\frac{1}{2}$ equal parts.

57. PRINCIPLES.—1. *Multiplying the numerator or dividing the denominator by a number multiplies the fraction by that number.*

2. *Dividing the numerator or multiplying the denominator by a number divides the fraction by that number.*

3. *Multiplying or dividing both numerator and denominator by the same number does not change the value of the fraction.*

EXERCISES.

58. *1.* Read the following fractions, and copy separately: 1, the simple fractions; 2, the proper fractions; 3, the improper fractions; 4, the mixed numbers; 5, the compound fractions; 6, the complex fractions:

$\frac{13}{16}$; $4\frac{1}{4}$; $\frac{11}{4}$; $\frac{5}{10}$; $\frac{7}{8}$; $\frac{11}{5}$ of $\frac{10}{17}$; $175\frac{3}{4}$; $\frac{3\frac{3}{4}}{5}$; $\frac{7}{8}$ of $3\frac{1}{3}$, $13\frac{2}{3}$; $\frac{7}{9}$; $\frac{16}{15}$; $7\frac{3}{4}$; $87\frac{7}{9}$; $46\frac{3}{5}$; $\frac{141}{8}$; $\frac{18}{25}$; $\frac{14\frac{1}{2}}{20}$; $\frac{2}{3}$; $\frac{3}{4}$ of $\frac{8}{9}$.

2. Write the following fractions: three fourths; seven eighths; nineteen sixteenths; five, and one half; one hundred and three thirty-seconds; one hundred, and three thirty-seconds; forty-eight, and five twelfths; eleven tenths; nine forty-fifths; thirty-six twenty-eighths; sixty-five forty-eighths.

3. Write the following fractions: eight ninths; thirteen, and two-thirds; sixteen twenty-fourths; ten tenths; fourteen, and forty-six hundredths; nineteen one hundred nineteenths; thirty-six four hundred thirty-seconds.

Reduction.

59. Reduction of Fractions is the changing their form without changing their value.

60. A fraction is reduced to *lower terms* when the numerator and denominator are expressed in smaller numbers.

61. A fraction is in its *lowest terms* when its numerator and denominator have no common divisor.

62. A fraction is reduced to *higher terms* when the numerator and denominator are expressed in larger numbers.

63. To reduce a fraction to its lowest terms.

Ex. Reduce $\frac{84}{126}$ to its lowest terms.

OPERATION.

$\frac{84}{126} = \frac{14}{21} = \frac{2}{3}$

ANALYSIS.—Dividing both terms of the fraction, $\frac{84}{126}$, by the common divisor, 6, the result is $\frac{14}{21}$; dividing both terms of $\frac{14}{21}$ by the common divisor, 7, the result is $\frac{2}{3}$. Since 2 and 3 have no common divisor, the fraction is reduced to its lowest terms (**61**).

The value of the fraction has not been changed, because both terms have been divided by the same number (**57**, 3).

The same result is often more readily obtained by dividing both terms by the greatest common divisor.

64. RULE.—*Divide the terms of the fraction by any number that will divide both without a remainder, and continue the operation with the resulting fractions until they have no common divisor.* Or,

Divide the terms of the fraction by their greatest common divisor.

EXAMPLES.

65. Reduce to their lowest terms,

1.	$\frac{32}{48}$.	*9.*	$\frac{96}{108}$.	*17.*	$\frac{648}{1024}$.	*25.*	$\frac{888}{912}$.
2.	$\frac{45}{60}$.	*10.*	$\frac{132}{168}$.	*18.*	$\frac{325}{450}$.	*26.*	$\frac{1056}{1584}$.
3.	$\frac{48}{64}$.	*11.*	$\frac{105}{120}$.	*19.*	$\frac{375}{1000}$.	*27.*	$\frac{1136}{3088}$.
4.	$\frac{63}{81}$.	*12.*	$\frac{144}{180}$.	*20.*	$\frac{625}{2000}$.	*28.*	$\frac{1278}{3474}$.
5.	$\frac{72}{128}$.	*13.*	$\frac{1296}{1416}$.	*21.*	$\frac{86}{946}$.	*29.*	$\frac{2638}{3072}$.
6.	$\frac{85}{105}$.	*14.*	$\frac{275}{625}$.	*22.*	$\frac{125}{625}$.	*30.*	$\frac{1024}{1728}$.
7.	$\frac{90}{135}$.	*15.*	$\frac{528}{1760}$.	*23.*	$\frac{125}{1000}$.	*31.*	$\frac{1001}{1144}$.
8.	$\frac{18}{144}$.	*16.*	$\frac{512}{728}$.	*24.*	$\frac{6125}{10000}$.	*32.*	$\frac{1440}{1728}$.

66. To reduce a fraction to higher terms.

Ex. Reduce $\frac{3}{4}$ to a fraction whose denominator is 32.

OPERATION.

$32 \div 4 = 8$

$\frac{3}{4} = \frac{24}{32}$

ANALYSIS.—The fraction $\frac{3}{4}$ is reduced to *thirty-seconds*, without changing its value, by multiplying the terms by the number that will cause its denominator 4 to become 32 (**57**, 3). By dividing the required denominator 32 by the given denominator 4, this number is found to be 8. Multiplying both terms of $\frac{3}{4}$ by 8, the result is $\frac{24}{32}$.

67. RULE.—*Divide the required denominator by the denominator of the given fraction, and multiply both terms of the given fraction by the quotient.*

EXAMPLES.

68. *1.* Reduce $\frac{3}{4}$ to 48ths.

2. Change $\frac{7}{12}$ to an equivalent fraction having 60 for its denominator.

3. Reduce $\frac{2}{3}$, $\frac{5}{6}$, $\frac{9}{16}$ each to 48ths.

4. Reduce $\frac{5}{7}$, $\frac{3}{5}$, $\frac{21}{15}$ each to 105ths.

5. Reduce $\frac{11}{14}$, $\frac{5}{8}$, $\frac{1}{2}$ each to 56ths.

6. Reduce $\frac{7}{16}$, $\frac{11}{12}$, $\frac{19}{32}$ each to 96ths.

7. Reduce $\frac{7}{8}$, $\frac{8}{9}$, $\frac{3}{10}$ each to 360ths.

8. Reduce $\frac{17}{24}$, $\frac{5}{8}$, $\frac{11}{18}$ each to 72ds.

9. Reduce $\frac{4}{9}$, $\frac{20}{27}$, $\frac{17}{12}$ each to 108ths.

10. Reduce $\frac{5}{6}$, $\frac{3}{8}$, $\frac{11}{30}$ each to 360ths.

69. To reduce two or more fractions to equivalent fractions having their least common denominator.

70. A **Common Denominator** of two or more fractions is a denominator to which they can all be reduced, and is the common multiple of their denominators.

71. The **Least Common Denominator** of two or more fractions is the least denominator to which they can be reduced, and is the least common multiple of their denominators.

Ex. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{9}{10}$ to equivalent fractions having their least common denominator.

$$\frac{2}{3} = \frac{40}{60}$$
$$\frac{3}{4} = \frac{45}{60}$$
$$\frac{5}{6} = \frac{50}{60}$$
$$\frac{9}{10} = \frac{54}{60}$$

OPERATION.

$$\begin{array}{r|l} 2 & \cancel{3},\ 4,\ 6,\ 10 \\ \hline & \quad 2\ \ 3\ \ \ 5 \end{array}$$

$$2 \times 2 \times 3 \times 5 = 60$$

ANALYSIS.—The least common multiple of the denominators is found to be 60 (**35**), which we take as the least common denominator. By Art. **67**, $\frac{2}{3}$ is reduced to $\frac{45}{60}$. We proceed in the same manner with each of the other fractions. The value of each fraction remains unchanged, since both terms have been multiplied by the same number. In many cases, the least common denominator can be readily found by inspection.

72. RULE.—*Find the least common multiple of the given denominators for the least common denominator, and reduce the given fractions to this denominator.*

EXAMPLES.

73. Reduce the following fractions to equivalent fractions having their least common denominator:

1. $\frac{2}{5}$, $\frac{7}{10}$, $\frac{4}{15}$.

2. $\frac{3}{3}$, $\frac{1}{2}$, $\frac{5}{7}$.

3. $\frac{11}{12}$, $\frac{8}{15}$, $\frac{27}{10}$.

4. $\frac{7}{8}$, $\frac{13}{6}$, $\frac{9}{16}$.

5. $\frac{20}{21}$, $\frac{25}{28}$, $\frac{11}{14}$.

6. $\frac{3}{5}$, $\frac{7}{16}$, $\frac{13}{20}$.

7. $\frac{4}{7}$, $\frac{13}{35}$, $\frac{4}{5}$.

8. $\frac{23}{24}$, $\frac{8}{9}$, $\frac{17}{18}$.

9. $\frac{17}{12}$, $\frac{13}{8}$, $\frac{10}{9}$.

10. $\frac{13}{20}$, $\frac{16}{25}$, $\frac{3}{10}$.

11. $\frac{13}{16}$, $\frac{17}{48}$, $\frac{47}{72}$.

12. $\frac{22}{45}$, $\frac{49}{75}$, $\frac{16}{25}$.

74. To reduce an integer or a mixed number to an improper fraction.

Ex. In 18 units, how many fourths?

OPERATION.

18
4
72 fourths.

ANALYSIS.—In 1 there are 4 fourths ($\frac{4}{4}$), and in 18, eighteen times 4 fourths, or 72 fourths ($\frac{72}{4}$). Hence, $18 = \frac{72}{4}$.

Ex. Reduce $16\frac{7}{8}$ to an improper fraction.

OPERATION.

$16\frac{7}{8}$
8
128 eighths.
7 eighths.
135 eighths.

ANALYSIS.—In 1 there are 8 eighths ($\frac{8}{8}$), and in 16, sixteen times 8 eighths, or 128 eighths ($\frac{128}{8}$). 128 eighths and 7 eighths are 135 eighths. Hence, $16\frac{7}{8} = \frac{135}{8}$.

75. RULE.—*Multiply the integer by the required denominator, and to the product add the numerator of the fraction, and under the result write the denominator.*

NOTE.—When the numerator of the fraction is a small number, add it mentally to the product of the integer and the denominator.

EXAMPLES.

76. *1.* In 27, how many ninths?

2. Reduce $46\frac{1}{2}$ to halves.

3. How many eighths of a peck in $37\frac{7}{8}$ pecks?

Reduce the following to improper fractions:

4. $37\frac{3}{4}$; $19\frac{7}{8}$; $208\frac{9}{16}$.

5. $56\frac{2}{3}$; $49\frac{5}{6}$; $182\frac{5}{7}$.

6. $375\frac{1}{2}$; $94\frac{9}{10}$; $46\frac{5}{8}$.

7. $44\frac{3}{4}$; $37\frac{5}{12}$; $191\frac{3}{16}$.

8. $12\frac{1}{2}$; $48\frac{7}{20}$; $45\frac{5}{12}$.

9. $81\frac{2}{3}$; $196\frac{1}{2}$; $375\frac{3}{4}$.

10. $116\frac{11}{25}$; $456\frac{4}{11}$; $871\frac{3}{36}$.

11. $24\frac{3}{14}$; $179\frac{13}{16}$; $1767\frac{3}{4}$.

12. $87\frac{3}{8}$; $490\frac{5}{12}$; $168\frac{11}{18}$.

13. $384\frac{5}{8}$; $161\frac{3}{4}$; $175\frac{25}{32}$.

77. To reduce an improper fraction to an integer or a mixed number.

Ex. Reduce $\frac{27}{4}$ to a mixed number.

ANALYSIS.—$1 = \frac{4}{4}$; hence in $\frac{27}{4}$, there are as many units as 4 fourths are contained times in 27 fourths, or $6\frac{3}{4}$.

78. RULE.—*Divide the numerator by the denominator.*

EXAMPLES.

79. *1.* Change $\frac{31}{8}$ to a mixed number.

2. Reduce $\frac{64}{4}$ of a dollar to dollars.

Reduce to integers or mixed numbers:

3. $\frac{375}{4}$; $\frac{416}{8}$.

4. $\frac{138}{2}$; $\frac{376}{3}$.

5. $\frac{518}{9}$; $\frac{444}{5}$.

6. $\frac{303}{16}$; $\frac{427}{12}$.

7. $\frac{542}{16}$; $\frac{873}{4}$.

8. $\frac{3861}{12}$; $\frac{1248}{16}$.

9. $\frac{445}{9}$; $\frac{785}{48}$.

10. $\frac{387}{16}$; $\frac{441}{32}$.

11. $\frac{511}{25}$; $\frac{987}{60}$.

12. $\frac{411}{15}$; $\frac{1385}{36}$.

13. $\frac{387}{16}$; $\frac{1416}{32}$.

14. $\frac{517}{8}$; $\frac{2387}{49}$.

ADDITION.

80. Addition of Fractions is the process of finding the sum of two or more fractions.

81. PRINCIPLE.—*In order that fractions may be added, they must have like denominators and be parts of like units.*

Ex. What is the sum of $\frac{5}{12}$, $\frac{9}{12}$, and $\frac{1}{12}$?

OPERATION.

$$\frac{5}{12} + \frac{9}{12} + \frac{1}{12} = \frac{15}{12} = \frac{5}{4} = 1\frac{1}{4}$$

ANALYSIS.—As these fractions have a common denominator, we add their numerators, and write their sum, 15, over the common denominator, 12. $\frac{15}{12} = 1\frac{1}{4}$, the required result.

Ex. Add $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$.

OPERATION.

$$\frac{2}{3} + \frac{3}{4} + \frac{5}{6} = \frac{8 + 9 + 10}{12} = \frac{27}{12} = 2\frac{3}{12} = 2\frac{1}{4}.$$

ANALYSIS.—Reduce the given fractions to equivalent fractions having the least common denominator, 12 (**72**). Then proceed as in previous example.

Ex. Find the sum of $29\frac{1}{6}$, $38\frac{3}{4}$, $17\frac{5}{8}$, and $42\frac{1}{3}$.

OPERATION.

$29\frac{1}{6}$	$\frac{4}{24}$
$38\frac{3}{4}$	$\frac{18}{24}$
$17\frac{5}{8}$	$\frac{15}{24}$
$42\frac{1}{3}$	$\frac{8}{24}$
$127\frac{7}{8}$	$\frac{45}{24} = 1\frac{7}{8}$

ANALYSIS.—The sum of the fractions is $\frac{45}{24} = 1\frac{7}{8}$, which added to the sum of the integers, gives $127\frac{7}{8}$, the required result.

Ex. How many yards in 12 pieces of prints containing $46\frac{1}{4}$, $48\frac{2}{4}$, $51\frac{2}{4}$, $49\frac{3}{4}$, $44\frac{1}{4}$, $48\frac{2}{4}$, $47\frac{1}{4}$, 49, $47\frac{3}{4}$, $50\frac{3}{4}$, $48\frac{1}{4}$, $48\frac{2}{4}$ yards respectively.

OPERATION.

$46\frac{1}{4}$	$47\frac{1}{4}$	
$48\frac{2}{4}$	49	
$51\frac{2}{4}$	$47\frac{3}{4}$	
$49\frac{3}{4}$	$50\frac{3}{4}$	
$44\frac{1}{4}$	$48\frac{1}{4}$	
$48\frac{2}{4}$	$48\frac{2}{4}$	$580\frac{1}{4}$.

ANALYSIS.—The sum of the fourths is $\frac{11}{4} = 5\frac{1}{4}$, which added to the sum of the integers gives $580\frac{1}{4}$, the total number of yards.

82. RULE.—*Reduce the given fractions to equivalent fractions having the least common denominator. Write the sum of the numerators over the common denominator, and reduce the resulting fraction to its simplest form.*

When there are mixed numbers or integers, add the integers and fractions separately, and then add the results.

EXAMPLES.

83. Add the following:

1. $\frac{5}{18}$, $1\frac{1}{18}$, $\frac{7}{18}$, and $1\frac{5}{18}$.
2. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$.
3. $12\frac{1}{2}$, $7\frac{3}{8}$, $16\frac{9}{16}$, and $38\frac{3}{4}$.
4. $48\frac{4}{9}$, $46\frac{7}{18}$, $31\frac{2}{3}$, and $17\frac{1}{6}$.
5. $127\frac{7}{48}$, $\frac{23}{32}$, $175\frac{3}{8}$, and $\frac{5}{6}$.
6. $141\frac{9}{20}$, $197\frac{3}{4}$, and $43\frac{7}{12}$.
7. $75\frac{2}{5}$, $\frac{5}{9}$, $1028\frac{2}{3}$, and $1\frac{3}{18}$.
8. $\frac{7}{8}$, $119\frac{1}{3}$, $240\frac{3}{4}$, and $17\frac{3}{16}$.
9. $46\frac{1}{4}$, $48\frac{3}{4}$, $40\frac{2}{4}$, 49, $47\frac{3}{4}$, and $46\frac{2}{4}$.
10. $40\frac{3}{4}$, $41\frac{1}{4}$, $48\frac{2}{4}$, $44\frac{1}{4}$, $49\frac{3}{4}$, $48\frac{2}{4}$, $49\frac{3}{4}$, $49\frac{1}{4}$, $47\frac{3}{4}$, $48\frac{3}{4}$, $48\frac{3}{4}$, and $49\frac{1}{4}$.
11. $18\frac{8}{9}$, $27\frac{1}{4}$, $42\frac{2}{3}$, and $51\frac{5}{8}$.
12. $146\frac{3}{4}$, $1\frac{4}{7}$, $53\frac{5}{14}$, and $68\frac{1}{2}$.
13. $1172\frac{5}{6}$, $19\frac{2}{3}$, $440\frac{7}{8}$, and $6\frac{3}{4}$.
14. $\frac{7}{16}$, $106\frac{5}{12}$, $37\frac{8}{9}$, and $7\frac{1}{8}$.
15. 175, $116\frac{7}{10}$, $143\frac{3}{5}$, and $27\frac{4}{7}$.
16. $20\frac{2}{5}$, $164\frac{3}{7}$, $1\frac{1}{21}$, and $43\frac{5}{9}$.
17. $44\frac{1}{3}$, $16\frac{7}{8}$, $29\frac{7}{16}$, and $13\frac{3}{4}$.
18. $31\frac{1}{4}$, $48\frac{3}{4}$, $62\frac{1}{4}$, $19\frac{3}{4}$, $27\frac{2}{4}$, $48\frac{1}{4}$, and $37\frac{3}{4}$.
19. $61\frac{3}{4}$, $48\frac{1}{4}$, $47\frac{3}{4}$, 48, $48\frac{2}{4}$, $49\frac{1}{4}$, and $45\frac{3}{4}$.
20. $19\frac{3}{8}$, $444\frac{5}{16}$, $737\frac{1}{4}$, and $385\frac{1}{3}$.

SUBTRACTION.

84. Subtraction of Fractions is the process of finding the difference between two fractions.

85. PRINCIPLE.—*In order that fractions may be subtracted, they must have like denominators and be parts of like units.*

Ex. From $\frac{8}{9}$ take $\frac{5}{9}$.

OPERATION.

$\frac{8}{9} - \frac{5}{9} = \frac{3}{9} = \frac{1}{3}$

ANALYSIS.—As these fractions have a common denominator, we take the difference of the numerators, and place it over the common denominator. $\frac{3}{9} = \frac{1}{3}$ is the result required.

Ex. What is the difference between $\frac{3}{4}$ and $\frac{2}{3}$?

OPERATION.

$\frac{3}{4} - \frac{2}{3} = \frac{9-8}{12} = \frac{1}{12}$

ANALYSIS.—Reduce the given fractions to equivalent fractions having the least common denominator (72). Then proceed as in the previous example.

Ex. From $176\frac{3}{8}$ subtract $89\frac{3}{4}$.

OPERATION.

$176\frac{3}{8}$ $\quad \frac{3}{8}$

$89\frac{3}{4}$ $\quad \frac{6}{8}$

$86\frac{5}{8}$

ANALYSIS. $\frac{6}{8}$ from $\frac{3}{8}$ we cannot take; we therefore take $1 = \frac{8}{8}$ from 176, leaving 175. $\frac{3}{8} + \frac{8}{8} = \frac{11}{8}$. $\frac{11}{8} - \frac{6}{8} = \frac{5}{8}$. $175 - 89 = 86$. $86 + \frac{5}{8} = 86\frac{5}{8}$.

86. RULE.—*Reduce the given fractions to equivalent fractions having the least common denominator. Write the difference of the numerators over the common denominator, and reduce the resulting fraction to its simplest form.*

When there are mixed numbers, subtract the integers and fractions separately, and add the results.

EXAMPLES.

87. Find the difference between

1. $\frac{3}{4}$ and $\frac{5}{9}$.
2. $\frac{7}{8}$ and $\frac{5}{12}$.
3. $\frac{2}{3}$ and $\frac{11}{24}$.
4. $2\frac{1}{2}$ and $1\frac{9}{16}$.
5. $\frac{7}{10}$ and $\frac{8}{15}$.
6. $\frac{4}{7}$ and $\frac{3}{5}$.
7. $1\frac{1}{3}$ and $\frac{5}{6}$.
8. $\frac{5}{9}$ and $\frac{4}{11}$.
9. 1 and $\frac{11}{16}$.

10. $17\frac{1}{2}$ and $9\frac{1}{4}$.
11. $175\frac{1}{4}$ and $86\frac{1}{2}$.
12. $138\frac{3}{8}$ and $17\frac{1}{4}$.
13. $149\frac{1}{8}$ and $18\frac{5}{8}$.
14. $416\frac{7}{8}$ and $49\frac{3}{4}$.
15. $512\frac{3}{4}$ and $53\frac{7}{8}$.
16. 100 and $13\frac{3}{4}$.
17. $116\frac{5}{6}$ and $48\frac{2}{3}$.
18. $381\frac{2}{3}$ and $17\frac{3}{4}$.
19. $157\frac{5}{8}$ and $19\frac{3}{5}$.
20. 118^{3} and 48^{2}.
21. $387\frac{3}{8}$ and $116\frac{3}{4}$.
22. $248\frac{5}{12}$ and $129\frac{1}{3}$.
23. $764\frac{3}{16}$ and $375\frac{1}{8}$.
24. $764\frac{1}{8}$ and $375\frac{3}{16}$.
25. $827\frac{1}{8}$ and $737\frac{2}{3}$.
26. $919\frac{3}{4}$ and $447\frac{5}{16}$.
27. 376^{1} and 287^{3}.
28. 445^{2} and 318^{3}.
29. 737^{3} and 438^{2}.
30. 648^{1} and 526^{3}.

MULTIPLICATION.

88. To multiply a fraction by an integer.

89. PRINCIPLE.—*Multiplying the numerator or dividing the denominator by a number multiplies the value of the fraction by that number* (**57**, 1).

Ex. What will 4 pounds of tea cost @ $\$\frac{7}{8}$ a pound?

OPERATIONS.

$$\frac{7}{8} \times 4 = \frac{7 \times 4}{8} = \frac{28}{8} = 3\frac{1}{2}$$

Or,

$$\frac{7}{8} \times 4 = \frac{7}{8 \div 2} = \frac{7}{2} = 3\frac{1}{2}$$

Or,

$$\frac{7}{\cancel{8}_{2}} \times \frac{\cancel{4}^{1}}{1} = \frac{7}{2} = 3\frac{1}{2}$$

ANALYSIS.—If 1 pound costs $\$\frac{7}{8}$, 4 pounds will cost 4 times $\$\frac{7}{8}$, or $\$\frac{28}{8}$, equal to $\$3\frac{1}{2}$. Hence, 4 pounds of tea @ $\$\frac{7}{8}$ will cost $\$3\frac{1}{2}$.

To multiply $\frac{7}{8}$ by 4, multiply the numerator 7 by 4, or divide the denominator 8 by 4; either operation will give $3\frac{1}{2}$, the required product (*Prin.*).

By cancellation (**38**), the operation is shortened, and the result is obtained in its lowest terms.

Multiplying the numerator, as in the first operation, increases the number of parts, their size remaining the same; dividing the denominator multiplies the fraction by increasing the size of the parts, their number remaining the same.

Ex. Multiply $123\frac{3}{4}$ by 9.

OPERATION.

$$\begin{array}{r} 123\frac{3}{4} \\ \underline{\quad 9 \quad} \\ 6\frac{3}{4} \\ 1107 \\ \hline 1113\frac{3}{4} \end{array}$$

ANALYSIS.—Multiply the fraction $\frac{3}{4}$ and the integer 123 separately, and add the products. In practice, when possible, add the products mentally; *e. g.*, 9 times $\frac{3}{4}$ are $\frac{27}{4}$, equal to $6\frac{3}{4}$. Write the $\frac{3}{4}$. 9 times 3 are 27, and 6 are 33. Write the 3, and proceed as in simple numbers.

Ex. Multiply $227\frac{3}{4}$ by 175.

OPERATIONS.

	Or,	
$227\frac{3}{4}$		$227\frac{3}{4}$
175		175
4) 525		$87\frac{1}{2}$
$131\frac{1}{4}$		$43\frac{3}{4}$
1135		1135
1589		1589
227		227
$39856\frac{1}{4}$		$39856\frac{1}{4}$

ANALYSIS.—As in preceding example.

Or, by aliquot parts, when the fractions are fourths, eighths, etc., the fractions generally used in commercial operations.

$\frac{3}{4} = \frac{1}{2} + \frac{1}{4}$ ($\frac{1}{2}$ of $\frac{1}{2}$).

$\frac{1}{2}$ of 175 = $87\frac{1}{2}$.

$\frac{1}{4}$ of 175, or $\frac{1}{2}$ of $87\frac{1}{2}$ = $43\frac{3}{4}$.

90. RULE.—*Multiply the numerator or divide the denominator of the fraction by the integer.*

When the multiplicand is a mixed number, multiply the fraction and integer separately, and add the results.

EXAMPLES.

91. *1.* Find the cost of 20 yards of silk at $\$\frac{7}{8}$ a yard.

2. How much grain in 12 bins, each containing $76\frac{7}{8}$ bushels?

3. If 1 man earns $\$\frac{7}{8}$ in 1 day, how much will 16 men earn in 26 days?

4. If a ton of hay cost $\$16\frac{3}{4}$, how much will 22 tons cost?

5. Required the cost of 60 yards of muslin at $35\frac{5}{8}$ cents a yard?

Multiply

6. $\frac{9}{16}$ by 7.
7. $\frac{11}{16}$ by 8.
8. $\frac{20}{21}$ by 3.
9. $110\frac{1}{2}$ by 12.
10. $117\frac{3}{4}$ by 16.
11. $248\frac{5}{8}$ by 3.
12. $146\frac{2}{3}$ by 3.
13. $197\frac{1}{8}$ by 7.
14. $420\frac{9}{16}$ by 8.
15. $384\frac{5}{8}$ by 12.
16. $375\frac{1}{2}$ by 48.
17. $412\frac{3}{8}$ by 47.
18. $148\frac{7}{8}$ by 40.
19. $412\frac{3}{7}$ by 89.
20. $775\frac{1}{8}$ by 65.
21. $119\frac{9}{16}$ by 20.
22. $772\frac{3}{4}$ by 17.
23. $338\frac{5}{8}$ by 30.
24. $550\frac{5}{9}$ by 27.
25. $643\frac{3}{4}$ by 121.
26. $875\frac{3}{8}$ by 234.
27. $916\frac{1}{2}$ by 275.
28. $2341\frac{1}{2}$ by 318.
29. $678\frac{2}{3}$ by 427.
30. $625\frac{3}{4}$ by 516.
31. $718\frac{1}{4}$ by 542.
32. $275\frac{3}{8}$ by 287.
33. $813\frac{5}{8}$ by 319.
34. $444\frac{7}{8}$ by 412.
35. $555\frac{5}{6}$ by 875.
36. $817\frac{3}{4}$ by 416.
37. $913\frac{1}{4}$ by 375.
38. $787\frac{3}{4}$ by 525.

92. To multiply an integer by a fraction, or to find a fractional part of an integer.

93. PRINCIPLE.—*Multiplying by a fraction is taking such part of the multiplicand as the fraction is of a unit.*

Ex. If 1 ton of hay cost \$18, what will $\frac{3}{4}$ of a ton cost?

OPERATIONS.

	Or,	Or,
4) 18	18	
$4\frac{1}{2}$	3	$\frac{3}{4}$ of $\overset{9}{\cancel{18}}_{1} = \frac{27}{2} = 13\frac{1}{2}$
3	4) 54	$\overset{2}{}$
$13\frac{1}{2}$	$13\frac{1}{2}$	

ANALYSIS.—If 1 ton cost \$18, $\frac{3}{4}$ of a ton will cost $\frac{3}{4}$ of \$18. $\frac{3}{4}$ of \$18 is 3 times $\frac{1}{4}$ of \$18. $\frac{1}{4}$ of \$18 is $\$4\frac{1}{2}$ (taking $\frac{1}{4}$ is the same as dividing by 4), and 3 times $\$4\frac{1}{2}$ is $\$13\frac{1}{2}$.

Or, $\frac{3}{4}$ of \$18 is $\frac{1}{4}$ of 3 times \$18. 3 times \$18 is \$54. $\frac{1}{4}$ of \$54 is $\$13\frac{1}{2}$.

Ex. Find the product of 175 and $8\frac{3}{4}$.

OPERATIONS.

	Or,	
175		175
$8\frac{3}{4}$		$8\frac{3}{4}$
4) 525		$43\frac{3}{4}$
$131\frac{1}{4}$		3
1400		$131\frac{1}{4}$
$1531\frac{1}{4}$		1400
		$1531\frac{1}{4}$

ANALYSIS.—Multiply by the fraction $\frac{3}{4}$ and by the integer 8 separately, and add the products.

Ex. Multiply 275 by $47\frac{3}{8}$.

FIRST OPERATION.	SECOND OPERATION.	THIRD OPERATION.
275	275	275
$47\frac{3}{8}$	$47\frac{3}{8}$	$47\frac{3}{8}$
8) 825	$34\frac{3}{8}$	$68\frac{3}{4}$
$103\frac{1}{8}$	3	$34\frac{3}{8}$
1925	$103\frac{1}{8}$	1925
1100	1925	1100
$13028\frac{1}{8}$	1100	$13028\frac{1}{8}$
	$13028\frac{1}{8}$	

ANALYSIS.—For the first and second operations, as in the preceding examples.

When the fractions are fourths, eighths, etc., multiply by means of aliquot parts.

$\frac{3}{8} = \frac{1}{4} + \frac{1}{8}$ ($\frac{1}{2}$ of $\frac{1}{4}$).

$\frac{1}{4}$ of 275 $= 68\frac{3}{4}$.

$\frac{1}{8}$ of 275, or $\frac{1}{2}$ of $68\frac{3}{4} = 34\frac{3}{8}$.

94. RULE.—*Multiply by the numerator of the fraction, and divide the product by the denominator.* Or,

Divide by the denominator of the fraction and multiply the quotient by the numerator.

When the multiplier is a mixed number, multiply by the fraction and integer separately, and add the results.

EXAMPLES.

95. *1.* Find the cost of $8\frac{3}{4}$ yds. of ribbon at 25 cts. a yard.

2. What is the cost of $42\frac{7}{8}$ pounds of butter at 26 cts. a pound.

3. Required the value of $48\frac{7}{8}$ yards of flannel at 75 cts. a yard.

Multiply

4. 84 by $\frac{3}{4}$.	*10.* 216 by $14\frac{3}{8}$.	*16.* 780 by $64\frac{5}{8}$.
5. 126 by $\frac{4}{7}$.	*11.* 375 by $24\frac{7}{8}$.	*17.* 512 by $37\frac{1}{4}$.
6. 49 by $\frac{5}{8}$.	*12.* 375 by $22\frac{2}{3}$.	*18.* 611 by $87\frac{1}{2}$.
7. 128 by $9\frac{1}{2}$.	*13.* 146 by $28\frac{3}{4}$.	*19.* 625 by $92\frac{3}{8}$.
8. 156 by $8\frac{1}{4}$.	*14.* 184 by $16\frac{4}{5}$.	*20.* 937 by $75\frac{3}{4}$.
9. 187 by $10\frac{3}{4}$.	*15.* 110 by $41\frac{7}{8}$.	*21.* 575 by $81\frac{7}{8}$.

96. To multiply a fraction by a fraction.*

Ex. At $\$\frac{7}{9}$ a pound, what will $\frac{3}{4}$ of a pound of tea cost?

OPERATION.

$\frac{3}{4} \times \frac{7}{9} = \frac{21}{36} = \frac{7}{12}$

Or, $\frac{\cancel{3}}{4} \times \frac{7}{\cancel{9}_{3}} = \frac{7}{12}$

ANALYSIS.—If 1 pound cost $\$\frac{7}{9}$, $\frac{3}{4}$ of a pound will cost $\frac{3}{4}$ of $\$\frac{7}{9}$. $\frac{3}{4}$ of $\$\frac{7}{9}$ is 3 times $\frac{1}{4}$ of $\$\frac{7}{9}$. $\frac{1}{4}$ of $\$\frac{7}{9}$ is $\$\frac{7}{36}$, and 3 times $\$\frac{7}{36}$ is $\$\frac{21}{36}$, or $\$\frac{7}{12}$.

Ex. What is the value of $8 \times 8\frac{1}{3} \times \frac{7}{10} \times \frac{1}{14}$?

OPERATION.

$\frac{\cancel{8}^{2}}{\cancel{1}} \times \frac{\cancel{25}^{5}}{\cancel{3}} \times \frac{\cancel{7}^{1}}{\cancel{10}_{\cancel{2}}} \times \frac{\cancel{1}^{1}}{\cancel{14}_{\cancel{2}}} = \frac{10}{3} = 3\frac{1}{3}$

ANALYSIS.—Reduce the integer 8 and the mixed number $8\frac{1}{3}$ to improper fractions, and multiply as in the preceding example.

97. RULE.—*Reduce integers and mixed numbers to improper fractions.*

Cancel all factors common to the numerators and denominators.

* The practical methods of multiplying one mixed number by another are given under Art. **108.**

Multiply the remaining numerators together for the numerator, and the remaining denominators for the denominator.

EXAMPLES.

98. Find the product of

1. $\frac{3}{5}$ and $\frac{2}{5}$.
2. $\frac{2}{3}$ and $\frac{3}{7}$.
3. $\frac{3}{4}$ and $\frac{5}{12}$.
4. $\frac{2}{5}$ and $\frac{10}{7}$.
5. $\frac{5}{8}$ and $\frac{12}{5}$.
6. 6, $3\frac{1}{3}$, and $\frac{4}{5}$.
7. $5\frac{2}{3}$, $\frac{6}{7}$, and $2\frac{1}{24}$.
8. $12\frac{1}{2}$, $10\frac{2}{3}$, and $\frac{5}{12}$.
9. $\frac{1}{3}$, $13\frac{1}{3}$, and $\frac{1}{5}$.
10. $26\frac{1}{4}$, $\frac{5}{7}$, and $\frac{2}{3}$.
11. $\frac{3}{4}$, $\frac{8}{9}$, and $16\frac{1}{7}$.
12. $13\frac{1}{3}$, $\frac{4}{5}$, and $\frac{7}{8}$.

Reduce the following compound fractions (**55**) to simple ones.

The word "of" is equivalent to the sign ×.

13. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$.
14. $\frac{2}{3}$ of $3\frac{1}{7}$ of $\frac{4}{5}$.
15. $\frac{3}{8}$ of $\frac{5}{9}$ of $7\frac{1}{2}$.
16. $\frac{3}{4}$ of $\frac{7}{8}$ of $5\frac{1}{3}$.
17. $\frac{5}{6}$ of $\frac{8}{9}$ of 18.
18. $\frac{3}{8}$ of $11\frac{2}{3}$ of $\frac{5}{7}$.
19. $\frac{16}{21}$ of $\frac{14}{15}$.
20. $\frac{7}{12}$ of $\frac{8}{9}$ of $\frac{5}{7}$.
21. $\frac{5}{8}$ of $\frac{40}{45}$ of $\frac{14}{15}$.
22. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{9}{5}$.
23. $\frac{1}{4}$ of $12\frac{1}{2}$ of $6\frac{2}{5}$.
24. $\frac{5}{6}$ of $\frac{18}{25}$ of $4\frac{1}{4}$.

Find the value of the following expressions:

25. $\frac{3}{8}$ of 1728.
26. $\frac{3}{4} \times 375$.
27. $\frac{7}{8}$ times 864.
28. $\frac{2}{3}$ of $75 \times \frac{3}{4}$ of $16\frac{2}{3}$.
29. $\frac{7}{8} \times \frac{3}{4}$ of $\frac{5}{12} \times \frac{8}{9}$.
30. $(\frac{4}{5} + \frac{9}{10}) \times (\frac{3}{7} + \frac{5}{14})$.
31. $(\frac{3}{4} - \frac{2}{3}) \times (\frac{7}{8} + \frac{3}{4})$.
32. $(\frac{5}{12} + \frac{3}{4}) \times (\frac{5}{14} - \frac{1}{7})$.
33. $37\frac{1}{3}$ times $\frac{2}{3}$ of $\frac{9}{10}$.
34. $\frac{3}{8}$ of $\frac{5}{6}$ by $\frac{5}{6}$ of $\frac{3}{5}$.

DIVISION.

99. To divide a fraction by an integer.

100. PRINCIPLE.—*Dividing the numerator or multiplying the denominator by a number divides the value of the fraction by that number* (**57**, 2).

Ex. What cost 1 pound of tea, if 5 pounds cost $\$3\frac{1}{3}$?

OPERATIONS.

$$\tfrac{10}{3} \div 5 = \frac{10 \div 5}{3} = \tfrac{2}{3}$$

$$\text{Or, } \tfrac{10}{3} \div 5 = \frac{10}{3 \times 5} = \tfrac{10}{15} = \tfrac{2}{3}$$

$$\text{Or, } \tfrac{\overset{2}{\cancel{10}}}{3} \times \tfrac{1}{\cancel{5}} = \tfrac{2}{3}$$

ANALYSIS.—If 5 pounds cost $\$3\frac{1}{3}$, 1 pound will cost $\frac{1}{5}$ of $\$3\frac{1}{3}$, or $\$\frac{2}{3}$.

To divide $\frac{10}{3}$ ($3\frac{1}{3}$) by 5, divide the numerator 10 by 5, or multiply the denominator 3 by 5; either operation will give $\frac{2}{3}$, the required quotient (*Prin.*).

Dividing the numerator, as in the first operation, decreases the number of parts, their size remaining the same; multiplying the denominator divides the fraction by decreasing the size of the parts, their number remaining the same.

Ex. Divide $867\frac{3}{4}$ by 4.

OPERATION.

$$\begin{array}{r|l} 4 & 867\frac{3}{4} \\ \hline & 216\frac{15}{16} \end{array} \qquad \begin{array}{l} 3\frac{3}{4} = \frac{15}{4} \\ \frac{15}{4} \div 4 = \frac{15}{16} \end{array}$$

ANALYSIS.—Dividing as in simple numbers, 4 is contained in $867\frac{3}{4}$, 216 times and a remainder of $3\frac{3}{4}$. $3\frac{3}{4}$ equals $\frac{15}{4}$, which divided by 4 is $\frac{15}{16}$.

101. RULE.—*Divide the numerator or multiply the denominator of the fraction by the integer.*

When the dividend is a mixed number, divide the integer and the fraction separately, and add the results.

EXAMPLES.

102. Divide

1. $\frac{3}{4}$ by 6.
2. $\frac{5}{6}$ by 3.
3. $\frac{8}{9}$ by 6.
4. $\frac{5}{12}$ by 4.
5. $\frac{8}{17}$ by 4.
6. $16\frac{3}{4}$ by 5.
7. $172\frac{1}{2}$ by 3.
8. $875\frac{2}{3}$ by 6.
9. $935\frac{3}{4}$ by 8.
10. $729\frac{1}{2}$ by 9.
11. $637\frac{1}{2}$ by 9.
12. $875\frac{5}{11}$ by 12.
13. $1716\frac{2}{3}$ by 8.
14. $1729\frac{1}{2}$ by 3.
15. $2418\frac{3}{4}$ by 5.
16. $3516\frac{3}{7}$ by 5.
17. $2428\frac{3}{4}$ by 3.
18. $6375\frac{3}{8}$ by 4.
19. $4287\frac{7}{8}$ by 2.
20. $3281\frac{1}{7}$ by 8.
21. $5316\frac{2}{3}$ by 4.
22. $7144\frac{1}{2}$ by 5.
23. $1729\frac{3}{4}$ by 3.
24. $1749\frac{1}{4}$ by 9.
25. $8763\frac{1}{2}$ by 6.
26. $7385\frac{3}{4}$ by 8.
27. $4255\frac{7}{8}$ by 9.
28. $7134\frac{3}{4}$ by 7.
29. $9727\frac{1}{5}$ by 12.
30. $6345\frac{3}{5}$ by 16.

103. To divide by a fraction.

104. The **Reciprocal** of a number is 1 divided by that number. Thus, the reciprocal of 4 is 1 divided by 4, or $\frac{1}{4}$.

The Reciprocal of a Fraction is 1 divided by that fraction.

105. PRINCIPLE. 1 *divided by a fraction is the fraction inverted.*

Thus, 1 divided by $\frac{3}{4}$ is $\frac{4}{3}$. This principle may be demonstrated as follows: In 1 there are 4 fourths. 1 fourth is contained in 4 fourths 4 times. Since $\frac{3}{4}$ is 3 times $\frac{1}{4}$, $\frac{3}{4}$ is contained in 1 $\frac{1}{3}$ as many times as $\frac{1}{4}$. Hence, $\frac{3}{4}$ is contained in 1 $\frac{1}{3}$ of 4 times, or $\frac{4}{3}$ times.

The reciprocal of a fraction is the fraction inverted.

Ex. At $\$\frac{3}{4}$ a yard, how many yards of cloth can be bought for \$5?

OPERATIONS.

$5 \div \frac{3}{4} = \frac{20}{4} \div \frac{3}{4} = 6\frac{2}{3}$

Or, $5 \div \frac{3}{4} = \frac{5}{1} \times \frac{4}{3} = \frac{20}{3} = 6\frac{2}{3}$

ANALYSIS.—Since 1 yard cost $\$\frac{3}{4}$, as many yards can be bought for \$5 as $\$\frac{3}{4}$ is contained times in \$5. 5 is equal to $\frac{20}{4}$, and 3 fourths is contained in 20 fourths $6\frac{2}{3}$ times.

Or, $\$\frac{3}{4}$ is contained in \$1 $\frac{4}{3}$ times (*Prin.*), and in \$5, 5 times $\frac{4}{3}$ or $\frac{20}{3}$, equal to $6\frac{2}{3}$ times.

Ex. At $\$\frac{3}{4}$ a yard, how many yards of cloth can be bought for $\$\frac{5}{6}$?

OPERATIONS.

$\frac{5}{6} \div \frac{3}{4} = \frac{10}{12} \div \frac{9}{12} = 1\frac{1}{9}$

Or, $\frac{5}{6} \div \frac{3}{4} = \frac{5}{6} \times \frac{4}{3} = \frac{20}{18} = 1\frac{1}{9}$

Or, $\frac{5}{6} \div \frac{3}{4} = \frac{\overset{2}{\cancel{5}}}{\underset{3}{\cancel{6}}} \times \frac{4}{3} = \frac{10}{9} = 1\frac{1}{9}$

ANALYSIS.—Since 1 yard cost $\$\frac{3}{4}$, as many yards can be bought for $\$\frac{5}{6}$ as $\$\frac{3}{4}$ is contained times in $\$\frac{5}{6}$. $\frac{3}{4}$ is equal to $\frac{9}{12}$, and $\frac{5}{6}$ is equal to $\frac{10}{12}$. $\frac{9}{12}$ is contained in $\frac{10}{12}$ $1\frac{1}{9}$ times.

Or, $\$\frac{3}{4}$ is contained in \$1 $\frac{4}{3}$ times (*Prin.*), and in $\$\frac{5}{6}$, $\frac{5}{6}$ times $\frac{4}{3}$ or $\frac{20}{18}$, equal to $1\frac{1}{9}$ times.

Ex. If $6\frac{2}{3}$ yards of cloth cost \$5, what will 1 yard cost?

OPERATIONS.

$5 \div \frac{20}{3} = (5 \div 20) \times 3 = \frac{3}{4}$

Or, $5 \div \frac{20}{3} = \frac{5}{1} \times \frac{3}{20} = \frac{15}{20} = \frac{3}{4}$

Or, $5 \div \frac{20}{3} = \frac{\overset{1}{\cancel{5}}}{1} \times \frac{3}{\underset{4}{\cancel{20}}} = \frac{3}{4}$

ANALYSIS. $6\frac{2}{3}$ yards are equal to $\frac{20}{3}$ yards. Since $\frac{20}{3}$ yards cost \$5, $\frac{1}{3}$ of a yard will cost $\frac{1}{20}$ of \$5 or $\$\frac{1}{4}$, and $\frac{3}{3}$ or 1 yard will cost 3 times $\$\frac{1}{4}$ or $\$\frac{3}{4}$.

Or, the price per yard equals the cost, divided by the quantity as an abstract number. 5 divided by $\frac{20}{3}$ equals 5 times 1 divided by $\frac{20}{3}$, or 5 times $\frac{3}{20}$ (*Prin.*), equal to $\frac{3}{4}$.

Ex. Divide $2195\frac{5}{6}$ by $175\frac{2}{3}$.

OPERATION.

$175\frac{2}{3}$) $2195\frac{5}{6}$

6 6

1054) 13175 ($12\frac{1}{2}$

1054

2635

2108

527

$\frac{527}{1054} = \frac{1}{2}$

ANALYSIS.—Reduce both divisor and dividend to improper fractions, and divide as in preceding example.

Or, multiplying both divisor and dividend by the same number does not affect the quotient. Multiply both divisor and dividend by 6, the least common denominator, and divide as in simple numbers.

106. RULE.—*Reduce the divisor and dividend to equivalent fractions having a common denominator, and divide the numerator of the dividend by the numerator of the divisor.* Or,

Invert the terms of the divisor and proceed as in multiplication.

In dividing mixed numbers, multiply both divisor and dividend by the least common denominator, and divide as in simple numbers.

EXAMPLES.

107. Divide

1. 1 by $\frac{7}{8}$.

2. 16 by $\frac{4}{5}$.

3. 28 by $\frac{3}{4}$.

4. 49 by $\frac{7}{9}$.

5. 88 by $\frac{3}{4}$.

6. $\frac{3}{4}$ by $\frac{7}{8}$.

7. $\frac{5}{6}$ by $\frac{2}{3}$.

8. $\frac{9}{10}$ by $\frac{3}{8}$.

9. $\frac{7}{12}$ by $\frac{5}{8}$.

10. $\frac{3}{4}$ by $\frac{2}{3}$.

11. 28 by $4\frac{1}{2}$.

12. 33 by $3\frac{2}{5}$.

13. 64 by $5\frac{2}{5}$.

14. 73 by $8\frac{1}{3}$.

15. 45 by $7\frac{3}{7}$.

16. $8\frac{1}{2}$ by $3\frac{3}{4}$.

17. $6\frac{2}{3}$ by $3\frac{1}{3}$.

18. $4\frac{4}{7}$ by $3\frac{2}{3}$.

19. $7\frac{1}{2}$ by $8\frac{1}{3}$.

20. $9\frac{1}{3}$ by $18\frac{1}{3}$.

21. 875 by $33\frac{1}{3}$.

22. 625 by $83\frac{1}{3}$.

23. 516 by $34\frac{2}{3}$.

24. 917 by $43\frac{2}{3}$.

25. 864 by $86\frac{2}{5}$.

26. 702 by $30\frac{4}{7}$.

27. 920 by $73\frac{3}{5}$.

28. 720 by $43\frac{1}{3}$.

29. 700 by $37\frac{1}{3}$.

30. 560 by $26\frac{1}{4}$.

31. $682\frac{1}{2}$ by $45\frac{1}{2}$.

32. $847\frac{7}{8}$ by $89\frac{1}{4}$.

33. 984^3 by 75^3.

34. 862^2 by 18^3.

35. 731^1 by 56^1.

36. $431\frac{1}{4}$ by $18\frac{3}{4}$.

37. $983\frac{1}{3}$ by $29\frac{1}{2}$.

38. $504\frac{1}{6}$ by $36\frac{2}{3}$.

39. $583\frac{1}{3}$ by $43\frac{3}{4}$.

Find the value of the following complex fractions (**56**) and expressions of division:

40. $\frac{5\frac{1}{16}}{9}$; $\frac{4\frac{2}{3}}{35}$; $\frac{24\frac{3}{4}}{36}$.

41. $\frac{3\frac{1}{8}}{40}$; $\frac{8\frac{2}{3}}{13}$; $\frac{16\frac{2}{3}}{20}$.

42. $\frac{5\frac{1}{2}}{7\frac{1}{3}}$; $\frac{\frac{3}{4}}{\frac{7}{8}}$; $\frac{\frac{9}{10}}{\frac{3}{5}}$.

43. $\frac{\frac{2}{3}\text{ of }\frac{3}{4}}{\frac{1}{3}\text{ of }2\frac{1}{4}}$; $\frac{\frac{1}{4}+3\frac{1}{2}}{5\frac{2}{3}-3\frac{1}{6}}$.

44. $\frac{18\frac{1}{2}\div 12\frac{1}{3}}{16\frac{1}{2}-15\frac{1}{8}}$; $\frac{12\frac{1}{2}\times 11\frac{1}{5}}{68\frac{3}{4}+1\frac{1}{4}}$.

45. $\frac{175\frac{3}{4}-16\frac{7}{8}}{187\frac{1}{8}-186\frac{7}{8}}$; $\frac{38\frac{2}{3}-30\frac{1}{3}}{16\frac{1}{3}+8\frac{2}{3}}$.

108. To multiply mixed numbers together.*

Ex. What cost $1016\frac{1}{2}$ pounds of cotton, at $12\frac{3}{8}$ cents per pound?

Instead of reducing the mixed numbers to improper fractions, use the following methods. The second method (by aliquot parts) is preferable, and is well adapted to commercial operations, in which the fractions are usually halves, fourths, eighths, etc.

In business transactions, it is customary to omit the fraction in the result, if it is less than $\frac{1}{2}$, and to add 1 to the cents if it is more than $\frac{1}{2}$. Unless otherwise stated, the exact answers will be given to examples.

FIRST OPERATION.

$1016\frac{1}{2}$
$.12\frac{3}{8}$
8) $3049\frac{1}{2}$
$381\frac{3}{16}$
12198
$125.79\frac{3}{16}$

ANALYSIS.—Multiply $1016\frac{1}{2}$ by the fraction $\frac{3}{8}$ by multiplying by the numerator 3 and dividing by the denominator 8 (**92**); then multiply $1016\frac{1}{2}$ by the integer 12 (**88**), and add the results.

SECOND OPERATION.

$1016\frac{1}{2}$
$.12\frac{3}{8}$
$254\frac{1}{8}$
$127\frac{1}{16}$
12198
$125.79\frac{3}{16}$

ANALYSIS. $\frac{3}{8}=\frac{1}{4}+\frac{1}{8}$. Multiply $1016\frac{1}{2}$ by $\frac{1}{4}$ by dividing by 4. Multiply $1016\frac{1}{2}$ by $\frac{1}{8}$ by taking $\frac{1}{8}$ of the $254\frac{1}{8}$, the product by $\frac{1}{4}$. Multiply $1016\frac{1}{2}$ by 12 (**88**), and add the results.

* The multiplication of mixed numbers is purposely put in this connection, as it appropriately comes here, a knowledge of division of fractions being a prerequisite to a fair understanding of the process.

EXAMPLES.

109. (*1.*)

$$\begin{array}{r} 1675\frac{1}{2} \\ 9\frac{1}{2} \\ \hline 837\frac{3}{4} \\ 15079\frac{1}{2} \\ \hline 15917\frac{1}{4} \end{array}$$

(*2.*)

$$\begin{array}{r} 1675\frac{1}{2} \\ 347\frac{3}{4}\ [\frac{1}{2}+\frac{1}{4}] \\ \hline 837\frac{3}{4}\ [\text{prod. by } \frac{1}{2}] \\ 418\frac{7}{8}\ [\text{prod. by } \frac{1}{4}\ (\frac{1}{2} \text{ of } \frac{1}{2})] \\ 173\frac{1}{2} \\ 11725 \\ 6700 \\ 5025 \\ \hline 582655\frac{1}{8} \end{array}$$

Or,

$$\begin{array}{r} 1675\frac{1}{2} \\ 347\frac{3}{4} \\ \hline 4\,)\,5026\frac{1}{2} \\ 1256\frac{5}{8} \\ 173\frac{1}{2} \\ 11725 \\ 6700 \\ 5025 \\ \hline 582655\frac{1}{8} \end{array}$$

(*3.*)

$$\begin{array}{r} 864\frac{3}{4}\ [\frac{1}{2}+\frac{1}{4}] \\ 126\frac{5}{8}\ [\frac{1}{2}+\frac{1}{8}] \\ \hline 432\frac{3}{8}\ [\text{prod. by } \frac{1}{2}] \\ 108\frac{3}{32}\ [\text{prod. by } \frac{1}{8}\ (\frac{1}{4} \text{ of } \frac{1}{2})] \\ 63\ [126\times\frac{1}{2}] \\ 31\frac{1}{2}\ [126\times\frac{1}{4}\ (\frac{1}{2} \text{ of } \frac{1}{2})] \\ 5184 \\ 10368 \\ \hline 109498\frac{31}{32} \end{array}$$

Or,

$$\begin{array}{r} 864\frac{3}{4} \\ 126\frac{5}{8} \\ \hline 8\,)\,4323\frac{3}{4} \\ 540\frac{15}{32} \\ 94\frac{1}{2} \\ 5184 \\ 10368 \\ \hline 109498\frac{31}{32} \end{array}$$

4)378

Multiply in like manner,

4. $875\frac{1}{2}$ by $8\frac{1}{2}$; by $37\frac{1}{4}$; by $26\frac{3}{4}$.

5. $737\frac{1}{4}$ by $10\frac{1}{2}$; by $12\frac{1}{4}$; by $44\frac{3}{8}$.

6. $512\frac{3}{4}$ by $7\frac{1}{2}$; by $27\frac{1}{2}$; by $64\frac{3}{4}$.

7. $449\frac{3}{8}$ by $16\frac{1}{4}$; by $36\frac{1}{8}$; by $45\frac{3}{8}$.

8. $1612\frac{1}{4}$ by $13\frac{1}{2}$; by $42\frac{5}{8}$; by $185\frac{3}{4}$.

9. $2437\frac{3}{4}$ by $16\frac{1}{8}$; by $12\frac{1}{8}$; by $14\frac{7}{16}$.

REVIEW EXAMPLES.

110. *1.* Reduce $\frac{888}{1152}$ to its lowest terms.

2. Reduce $\frac{7}{8}$ to forty-eighths.

3. Reduce $727\frac{3}{8}$ to an improper fraction.

4. Reduce $\frac{13761}{8}$ to a mixed number.

5. Add $17\frac{1}{2}$, $37\frac{3}{4}$, $18\frac{2}{3}$, $49\frac{5}{6}$, $13\frac{3}{8}$, and $56\frac{5}{12}$.

6. From $1728\frac{1}{8}$ take $865\frac{3}{4}$.

7. Multiply $\frac{1}{2} \times 3\frac{1}{2} \times \frac{6}{14} \times \frac{3}{10} \times 16\frac{2}{3}$.

8. Multiply $1727\frac{3}{4}$ by 175.

9. Multiply 1727 by $175\frac{3}{4}$.

10. Divide $1\frac{7}{8}$ by $\frac{3}{16}$.

11. Divide 1736 by $144\frac{2}{3}$.

12. Divide $5779\frac{2}{3}$ by $275\frac{2}{3}$.

13. Divide $12346\frac{1}{4}$ by 7; by 35.

14. What is the cost of 1583 pounds sugar @ $11\frac{3}{4}$ cts. per pound?

15. Add $\frac{3}{8}$ of $\frac{7}{8}$ of $4\frac{1}{6}$, $\frac{5}{8}$, $136\frac{2}{3}$, and $\frac{5\frac{3}{5}}{\frac{7}{8}}$.

16. A merchant sold a quantity of goods for \$7344, which was $\frac{3}{4}$ of the cost. What was the loss?

17. Required the value of 2993 pounds of sugar @ $9\frac{3}{8}$ cts. per pound?

18. If $\frac{7}{8}$ of a ship is worth $\$424301\frac{1}{2}$, what is the value of the whole?

19. Bought $47\frac{3}{4}$ yards of cloth at $\$4\frac{1}{2}$ per yard, and paid for it in wheat at $\$2\frac{1}{4}$ per bushel; how many bushels were required?

20. Find the value of $31\frac{1}{16}\frac{3}{8}$ pounds snuff @ 72 cts. per pound.

21. The less of two numbers is $777\frac{3}{4}$ and their difference $117\frac{3}{8}$; what is the greater number?

22. A and B together have \$1728; if A's money is equal to $\frac{5}{7}$ of B's, how much have each?

23. A merchant having $2146\frac{3}{4}$ yards of cloth, sold $\frac{5}{8}$ of it at $\$1\frac{3}{4}$ a yard, and the remainder at $\$2\frac{1}{2}$ a yard; how much did he receive?

24. A number being increased by $\frac{5}{8}$ of itself, the sum is 546; what is the number?

25. A man had \$5280; he bought goods with $\frac{3}{8}$ of it, and then lent $\frac{1}{4}$ of the balance to a friend; how much had he left?

26. Find the selling price of goods sold at a profit of \$75, being $\frac{2}{5}$ of the cost.

27. Mr. A bought $117\frac{3}{4}$ acres of land at one time, and $87\frac{5}{8}$ at another; after selling $110\frac{7}{8}$ acres, how much remained?

28. If $8\frac{3}{4}$ tons of coal cost $\$30\frac{5}{8}$, what will $27\frac{1}{2}$ tons cost? How many tons can be bought for $\$127\frac{3}{4}$?

29. A man paid $\$1145\frac{5}{8}$ for a horse and carriage. What was the value of each, the carriage being valued at $\frac{5}{8}$ as much as the horse?

30. If $\frac{3}{4}$ of a farm is valued at $\$2253\frac{1}{2}$, what is the value of $\frac{2}{3}$ of it?

31. What is the value of 2102^1 yards prints at 7^2 cents per yard?

32. What number must be taken from $96\frac{3}{4}$, and the remainder multiplied by $16\frac{2}{3}$, that the product shall be $770\frac{5}{6}$?

33. What is the value of 164^2 yards muslin at $5\frac{3}{4}$ cents per yard?

34. If 7 barrels of oil contain $313\frac{1}{4}$ gallons, how many gallons will $2\frac{3}{4}$ barrels contain?

35. An executor collects $\$12724.84$. He pays out $\$4096.48$ and the residue he disburses to the widow and her four children as follows: The widow receives a third part, and the remainder is divided equally among the children. What was the share of each?

36. What number increased by $\frac{3}{4}$ of itself will produce $2456\frac{1}{8}$?

37. Find the selling-price of goods, bought at $\$144$, and sold at $\frac{1}{3}$ above cost.

38. A invests $\frac{5}{8}$ of his capital in real estate, and has $\$1725$ remaining; what is his capital?

39. Bought a barrel of sugar containing 218 lbs., at $8\frac{1}{2}$ cents per pound. During the sale, it dried away $\frac{1}{10}$. Did I gain or lose, and how much, by selling it at $9\frac{1}{4}$ cents per pound?

40. Multiply $2375\frac{1}{2}$ by $8\frac{1}{2}$; by $10\frac{1}{4}$.

41. Multiply $1727\frac{3}{4}$ by $18\frac{1}{2}$; by $107\frac{3}{8}$.

42. Multiply $377\frac{1}{4}$ by $16\frac{1}{2}$; by $37\frac{3}{4}$.

43. Multiply $875\frac{1}{2}$ by $22\frac{1}{4}$; by $9\frac{5}{8}$.

44. A merchant sold $12\frac{7}{8}$ yards of silk to one customer, $21\frac{3}{4}$ to another, $20\frac{3}{8}$ to another, and $28\frac{1}{2}$ to another; at $\$2\frac{3}{8}$ per yard, how many dollars did he receive?

45. An army loses $\frac{3}{16}$ of its number in battle and has 16042 remaining; how many did it originally contain?

46. What is the cost of 34 pieces prints, containing 1604^2 yards, at 5^1 cents per yard?

47. What is the value of 12 pieces prints containing 48, 48^1, 48^2, 48, 49^2, 48^3, 48, 49^3, 49^2, 48^3, 49^2, 48^3 yards respectively at 4^3 cents per yard?

48. A merchant purchased 29 pieces prints containing 48^3, 48^2, 41^2, 48^2, 48^3, 47, 49, 49^2, 52^1, 57^3, 48^3, 48^2, 38, 48^2, 48^2, 48^2, 47^3, 48^2, 48, 51, 48, 44^1, 51^2, 48, 42^3, 46^2, 48, 48^2, 48^3 yards respectively; what was the cost at 5^2 cents per yard?

49. There are 5280 feet in one mile, and $16\frac{1}{2}$ feet in one rod; how many rods in one mile?

50. A can do a certain piece of work in 10 days, and B can do it in 15 days; how long will it take them both to do it?

51. A market-woman bought 120 oranges at the rate of 5 for 2 cents, and sold $\frac{1}{2}$ of them at the rate of 3 for 1 cent, and the remainder at the rate of 2 for 1 cent. Did she gain or lose, and how much?

52. What is the duty on 22375 pounds sugar, at $2\frac{13}{16}$ cts. per pound?

53. A farmer sold $1276\frac{19}{32}$ bushels oats at 44 cts. per bushel, $876\frac{29}{56}$ bushels corn at $52\frac{3}{4}$ cts., and $3381\frac{48}{80}$ bushels wheat at \$1.32; how much did he receive?

54. How many bushels of corn at $54\frac{1}{4}$ cts. per bushel must a farmer exchange for 62 yards of sheeting at $8\frac{3}{4}$ cts. per yard, and 31 yards broadcloth at \$1.75 per yard?

55. What is the value of 45^3 yards damask at 77^2 cts. per yard?

56. The salary of the President of the United States is \$50000 per year; how much is that per day?

57. $1\frac{4}{16}$ pounds of beef and $1\frac{6}{16}$ pounds of flour are allowed to ration; how much will 617 rations cost, if the price of beef is $11\frac{3}{8}$ cts. per pound, and of flour $3\frac{1}{4}$ cts. per pound?

58. What is the value of 36385 pounds of corn at $48\frac{3}{4}$ cents per bushel, each bushel containing 56 pounds?

59. Foreign immigration since 1870, by fiscal years:

Years.	Number.	Years.	Number.	Years.	Number.
1870.....	356,303	1874.....	260,814	1878.....	138,469
1871.....	346,938	1875.....	191,231	1879.....	177,826
1872.....	437,750	1876.....	237,991	1880.....	457,257
1873.....	422,545	1877.....	141,857	1881.....	669,439

According to the above table, what was the average immigration per year? What per month?

DECIMALS.

DEFINITIONS.

111. A **Decimal** (from the Latin *decem*, ten) **Fraction** is a fraction whose denominator is 1 followed by one or more ciphers; as $\frac{3}{10}$, $\frac{16}{100}$, $\frac{7}{1000}$.

112. Decimal fractions arise from dividing a unit into 10 equal parts, and then dividing these parts into 10 other equal parts, and so on.

Thus, if a unit be divided into 10 equal parts, each part is called a *tenth*. If a unit be divided into 100 equal parts, or 1 tenth into 10 equal parts, the parts are called *hundredths*. If a unit be divided into 1000 equal parts, or 1 hundredth into 10 equal parts, the parts are called *thousandths*.

113. All the rules, principles, operations, etc., of common fractions may be applied to decimal fractions. Since decimal fractions increase and decrease uniformly according to the scale of ten, a more simple notation, similar to that of integers, has been devised for them.

A hundred is written 100; a tenth part of a hundred (ten) is written 10, the 1 being written one place to the right; a tenth part of one ten (one unit) is written 1, the 1 being written one place to the right; in like manner, a tenth part of one unit (one-tenth) is written .1, the 1 being written one place to the right; the tenth part of one-tenth (one hundredth) is written .01, the 1 being written one place to the right, etc., etc.

Decimal fractions, like integers, decrease from left to right in a tenfold ratio, and increase from right to left in the same ratio.

114. In the decimal notation, the numerator only is written, the denominator being indicated by the position of a point (.) called the *decimal point*. The decimal point separates the integral from the fractional part.

115. The denominator of a decimal fraction is understood, and is 1 with as many ciphers annexed as there are figures in the decimal; thus,

Form of common fraction.		Form of decimal fraction.			
$\frac{7}{10}$	is written	.7	and is read	seven tenths.
$\frac{8}{100}$	" "	.08	" "	eight hundredths.
$\frac{16}{1000}$	" "	.016	" "	sixteen thousandths.

Hereafter, the first form, that of the common fraction, will be called a *fraction*, and the second, that of the decimal notation, a *decimal*.

116. The first place to the right of the point is called *tenths*, the second place *hundredths*, the third place *thousandths*, and so on.

117. The relation between integers and decimals is shown in the following

NUMERATION TABLE.

etc., etc.	*Billions.*	Hundred-Millions.	Ten-Millions.	*Millions.*	Hundred-Thousands.	Ten-thousands.	*Thousands.*	Hundreds.	Tens.	Units.	Decimal Point.	Tenths.	Hundredths.	*Thousandths.*	Ten-thousandths.	Hundred-Thousandths.	*Millionths.*	Ten-Millionths.	Hundred-Millionths.	*Billionths.*	etc., etc.
	2	4	3	6	8	0	7	5	9	3	.	6	8	9	4	6	0	5	8	2	
	10th.	9th.	8th.	7th.	6th.	5th.	4th.	3d.	2d.	1st.		1st.	2d.	3d.	4th.	5th.	6th.	7th.	8th.	9th.	
	Orders of Integers.											Orders of Decimals.									

118. In the above table, observe that the first place to the left of units is called *tens*, and the first place to the right, *tenths*; the second place to the left of units is called *hundreds*, and the second place to the right, *hundredths*, etc. Hence the number of any order or place of the decimal, counting from the point, or from units' place, is the same as the number of ciphers in the denominator of the decimal.

119. A **Complex Decimal** has a fraction in its right-hand place.

Thus, $.16\frac{2}{3}$ $\left(\frac{16\frac{2}{3}}{100}\right)$ is a complex decimal, and is read $16\frac{2}{3}$ hundredths, the fraction not being counted as a decimal place.

120. PRINCIPLES.—1. *Annexing ciphers to a decimal does not alter its value.*

Annexing a cipher multiplies both the numerator and denominator by 10, and hence does not alter the value of the decimal (**57**, 3). Thus, .7 ($\frac{7}{10}$) = .70 ($\frac{70}{100}$) = .700 ($\frac{700}{1000}$).

2. *Each removal of the decimal point one place to the right multiplies the value of the decimal by 10.*

Removing the point one place to the right does not change the numerator, but divides the denominator by 10, and hence multiplies the value of the decimal (**57**, 1). Thus, .072 ($\frac{72}{1000}$) becomes .72 ($\frac{72}{100}$); $\frac{72}{100} = \frac{72}{1000} \times 10$.

3. *Each removal of the decimal point one place to the left divides the value of the decimal by 10.*

Removing the point one place to the left does not change the numerator, but multiplies the denominator by 10, and hence divides the value of the fraction by 10 (**57**, 2). Thus, .72 ($\frac{72}{100}$) becomes .072 ($\frac{72}{1000}$); $\frac{72}{1000} = \frac{72}{100} \div 10$.

NUMERATION OF DECIMALS.

121. RULE.—*Read the decimal as if it were an integer, and give it the name of its right-hand order.*

EXERCISES.

122. Write in words, or read orally the following numbers:

1.	.6.	*8.*	17.6.	*15.*	375.18$\frac{3}{4}$.
2.	.008.	*9.*	8.029.	*16.*	19.0033$\frac{1}{3}$.
3.	.27.	*10.*	24.000488.	*17.*	6.148$\frac{2}{3}$.
4.	.0375.	*11.*	400.000088.	*18.*	648.6$\frac{2}{3}$.
5.	.0108.	*12.*	76.7071.	*19.*	347.18005.
6.	.775.	*13.*	3000.0045.	*20.*	808.008.
7.	.1007.	*14.*	.3045.	*21.*	600.06.

NOTATION OF DECIMALS.

123. Write in the form of a decimal, sixty-four thousandths.

ANALYSIS.—Since there are only two figures in the numerator 64, and the right-hand figure of the decimal must occupy the third decimal place to express thousandths, it is necessary to prefix a cipher to bring the right-hand figure into its proper place. Therefore write *point, naught, six, four* (.064) in the order named.

124. RULE.—*Prefix the decimal point, and decimal ciphers if necessary, to the numerator written as an integer, so that the right-hand figure will occupy the order named.*

NOTE.—Before writing, determine mentally the place of the right-hand figure and the number of ciphers required. Write in all cases from left to right.

EXERCISES.

125. *1.* What is the name of the third decimal order? The sixth? The first? The fourth?

2. How many decimal places are required to express hundredths? Millionths? Ten-thousandths? Tenths? Hundred-millionths?

3. How many ciphers must be written after the decimal point in writing 375 millionths? 27 hundredths? 875 thousandths? 446 ten-millionths? 37 ten-thousandths?

4. Write the following as decimals, so that the decimal-points stand in the same vertical line: 8 tenths; 16 hundredths; 175 thousandths; 1804 millionths; 56 ten-thousandths; 3004 ten-millionths; 1728 ten-thousandths.

NOTE.—In the following exercises, the comma is used to separate the integral and decimal parts.

5. Seventeen, and seventy-five hundredths.

6. Twenty-six, and twenty-six thousandths.

7. Two hundred and forty-six ten-millionths.

8. Two hundred, and forty-six ten-millionths.

9. Three hundred and seventy-five, and eighteen hundred-thousandths.

10. Eight thousand, and sixty-five ten-thousandths.

11. Eight thousand and sixty-five ten-thousandths.

12. $\frac{7}{10}$, $\frac{37}{100}$, $19\frac{3}{10}$, $218\frac{325}{1000}$, $\frac{1728}{10000}$.

13. $16\frac{75}{100}$, $19\frac{37}{1000}$, $345\frac{12}{1000}$, $\frac{75}{10000}$, $\frac{1234}{10000}$.

14. $28\frac{16}{1000}$, $37\frac{27}{100}$, $376\frac{2008}{100000}$, $44\frac{1725}{10000}$, $\frac{6387}{100000}$.

15. $170\frac{1256}{100000}$, $16000\frac{225}{100000}$, $38\frac{3}{100}$, $\frac{16225}{100000}$.

16. $\frac{4}{10}$, $300\frac{75}{1000}$, $\frac{375}{1000}$, $1635\frac{216}{1000}$, $500\frac{63}{100}$.

17. $1027\frac{13}{100}$, $\frac{16}{10000}$, $\frac{214}{100000}$, $387\frac{1012}{10000000}$.

REDUCTION.

126. To reduce a fraction to a decimal.

Ex. Reduce $\frac{3}{4}$ to a decimal.

OPERATION.

$$\begin{array}{r} 4\,)\,3.00 \\ \hline .75 \end{array}$$

ANALYSIS. $\frac{3}{4}$ equals $\frac{1}{4}$ of 3 units. 3 units equal 300 hundredths. $\frac{1}{4}$ of 300 hundredths equal 75 hundredths.

127. RULE.—*Annex decimal ciphers to the numerator, and divide by the denominator, pointing off as many decimal places in the quotient as there are ciphers annexed.*

128. A fraction in its lowest terms can be reduced to a pure decimal only when its denominator contains no prime factors but 2 and 5. If the denominator or divisor contain any prime factor other than 2 and 5, the division will not end. The decimals thus produced are called **Interminate** or **Repeating Decimals,** and the figures repeated, **Repetends.**

When a fraction is in its lowest terms, its numerator and denominator have no common factors (**61**). Annexing ciphers to the numerator introduces the factors 2 and 5 only; hence, if the denominator is an exact divisor of the numerator with the ciphers annexed, it must contain these prime factors and none others.

EXAMPLES.

129. Reduce to equivalent decimals:

1. $\frac{1}{2}$.	*4.* $\frac{3}{8}$.	*7.* $\frac{17}{40}$.	*10.* $\frac{7}{12}$.	*13.* $16\frac{5}{8}$.
2. $\frac{7}{8}$.	*5.* $\frac{7}{16}$.	*8.* $\frac{2}{3}$.	*11.* $\frac{5}{7}$.	*14.* $27\frac{12}{13}$.
3. $\frac{3}{4}$.	*6.* $\frac{25}{32}$.	*9.* $\frac{5}{6}$.	*12.* $\frac{4}{9}$.	*15.* $36\frac{23}{24}$.

130. To reduce a decimal to a fraction.

Ex. Reduce .075 to an equivalent fraction.

OPERATION.

$$.075 = \frac{75}{1000} = \frac{3}{40}$$

ANALYSIS.—A decimal is changed to a fraction by writing its denominator, and omitting the decimal point and prefixed ciphers. $\frac{75}{1000} = \frac{3}{40}$ (**63**).

Ex. Change $.83\frac{1}{3}$ to a simple fraction.

OPERATION.

$$.83\frac{1}{3} = \frac{83\frac{1}{3}}{100} = \frac{83\frac{1}{3} \times 3}{100 \times 3} = \frac{250}{300} = \frac{5}{6}$$

ANALYSIS.—Reduce the complex fraction $\frac{83\frac{1}{3}}{100}$ to a simple fraction by multiplying both terms by the denominator 3. (**57**, 3.)

131. RULE.—*Omit the decimal point, supply the proper denominator, and reduce the fraction to its lowest terms.*

EXAMPLES.

132. Reduce to equivalent fractions:

1. .25.	*8.* .128.	*15.* $.33\frac{1}{3}$.	*22.* $.44\frac{4}{9}$.
2. .75.	*9.* .00144.	*16.* $.41\frac{2}{3}$.	*23.* $.1428571\frac{1}{7}$.
3. .375.	*10.* .512.	*17.* $.066\frac{2}{3}$.	*24.* $.08333\frac{1}{3}$.
4. .625.	*11.* .5625.	*18.* $.37\frac{1}{2}$.	*25.* 28.0375.
5. .875.	*12.* .1875.	*19.* $.104\frac{1}{6}$.	*26.* $107.166\frac{2}{3}$.
6. .125.	*13.* $.12\frac{1}{2}$.	*20.* $.097\frac{2}{3}$.	*27.* 175.096.
7. .016.	*14.* $.16\frac{2}{3}$.	*21.* $.0053\frac{1}{4}$.	*28.* 6.0175.

ADDITION.

133. Since decimals, like integers, increase and decrease uniformly according to a scale of ten, with the exception of placing the decimal point in the result (usually called *pointing off*), they may be *added, subtracted, multiplied,* and *divided* in the same manner as integers.

Ex. What is the sum of 28.7, 175.28, .037, 25.0045, and 4.08?

OPERATION.

28.7
175.28
.037
25.0045
4.08
233.1015

ANALYSIS.—Write the numbers so that units of the same order stand in the same column.

If the decimal points are in the same vertical line, tenths will necessarily be under tenths, hundredths under hundredths, etc. Add as in integers, and place the point in the result directly under the points of the numbers.

Ex. Add .6, .37¾, 16.048⅓, 8.1234⅔, and 24.125.

OPERATION.

.6	=	.6
.37¾	=	.3775
16.048⅓	=	16.0483⅓
8.1234⅔	=	8.1234⅔
24.125	=	24.125
		49.2742 1⅓

ANALYSIS.—Reduce the complex decimals as far as the decimal places extend in the other numbers. Since the fractions now express parts of the same fractional unit, they may be added.

In practice, the fractions may be rejected if the decimals are carried one place, at least, farther than accuracy is required.

134. RULE.—*Write the numbers so that their decimal points are in the same vertical line. Add as in integers, and place the decimal point in the result directly under the points in the numbers added.*

EXAMPLES.

135. *1.* Add ninety-seven hundredths; three hundred and forty-seven thousandths; sixteen, and seventy-five hundred-thousandths; four hundred and seventy-five, and two thousand and thirty-seven millionths.

2. Add four, and eighty-one thousandths; thirty-seven, and two hundred and one ten-thousandths; seven thousand and eight hundred-thousandths; seven thousand, and eight hundred-thousandths; nineteen hundredths; three hundred and sixty-four, and nine tenths; and fifty-six, and fifty-four thousandths.

3. Add three hundred and seventy-five, and eight hundredths; eighteen thousandths; ninety-six, and eighty-four hundredths; four, and four tenths; and eight hundred and seven ten-millionths.

4. What is the sum of 18 hundredths; 716 hundred-thousandths; 6342 millionths; 11567 ten-millionths; 625 ten-thousandths; 9 tenths; 99 hundredths; and 512 thousandths?

5. Add 81.86; 12.593; 4.004; 18.00129; .443; 400.043; .12875; 175.00175; 17.3008; 9000.0016; and .9016.

6. Required, the sum of 99 ten-thousandths; 157½ thousandths; 789¾ millionths; 6 tenths; 18¾ hundredths; 1728 ten-millionths; and 88 hundredths.

7. Add $1728.64; $0.37½; $18.44½; $10.18¾; $6.25; and $0.16¼.

8. What is the sum of $12.37\frac{1}{2}$; $144.18\frac{3}{4}$; $6.62\frac{1}{2}$; $175.06\frac{1}{4}$; $40.17\frac{7}{8}$; and 398?

9. Add $.1264\frac{1}{3}$; 12.875; 187.25; $9.1414\frac{5}{6}$; .12; $5.7604\frac{7}{12}$; and $.0008\frac{3}{5}$.

10. Add $.26\frac{1}{2}$; $4.18\frac{3}{4}$; $.0017\frac{3}{8}$; $.00864\frac{1}{3}$; $.04\frac{3}{5}$; $17.387\frac{1}{3}$; and .0102075.

SUBTRACTION.

136. Ex. From 12.75 subtract 8.125.

OPERATION.

12.75
8.125
4.625

ANALYSIS.—Write the subtrahend under the minuend so that units of the same order stand in the same column. Subtract as in integers, and place the point in the result directly under the points of the numbers.

If, as in this example, the minuend has not as many decimal places as the subtrahend, suppose decimal ciphers to be annexed until the right-hand figures are of the same order. (**120**.)

Reduce complex decimals as in addition (**133**).

137. RULE.—*Write the numbers so that their decimal points are in the same vertical line. Subtract as in integers, and place the point in the remainder directly under the points in the minuend and subtrahend.*

EXAMPLES.

138. *1.* From four, and sixty-five thousandths, subtract eight hundred and forty-seven ten-thousandths.

2. From twenty-seven hundredths take twenty-nine hundred-thousandths.

3. From nine thousand, and thirty-four ten-thousandths, subtract nine thousand and thirty-four ten-thousandths.

Find the difference between

4. 8.3644 and 7.8996.
5. 17.4586 and .785.
6. 1.010101 and .999999.
7. $173.46 and $87.29.
8. 3 and .873845.
9. $17.24\frac{1}{2}$ and $18.973\frac{1}{4}$.
10. $510.60 and $389.45\frac{1}{2}$.
11. $1728 and $.06\frac{3}{4}$.
12. $17.864\frac{3}{5}$ and 16.94.
13. $144.43\frac{1}{3}$ and 113.3875.
14. $54.37\frac{5}{6}$ and $.98\frac{3}{5}$.
15. $117.48\frac{3}{4}$ and $49.43\frac{5}{6}$.
16. $448.987\frac{1}{3}$ and $389.28\frac{4}{5}$.
17. $5556.8\frac{1}{7}$ and 44.48.
18. $968.44\frac{4}{9}$ and $37.386\frac{3}{5}$.
19. $49.45\frac{1}{3}$ and $48.9876\frac{1}{8}$.

MULTIPLICATION.

139. Ex. Multiply .144 by .12.

OPERATION.

.144
.12
.01728

ANALYSIS.—For convenience, write the right-hand figures of the factors in the same vertical line.

$.144 \times .12 = \frac{144}{1000} \times \frac{12}{100} = \frac{1728}{100000}$. Multiply the numerators of the two factors for the numerator of the product, as in multiplication of fractions. In the above multiplication of fractions, it will be observed that the number of ciphers in the denominator of the product equals the sum of the ciphers in the denominators of the two factors. Since each cipher represents a decimal place, the product should have as many decimal places as both factors.

If the number of figures in the product is less than the number of decimal places in the two factors, supply the deficiency by prefixing ciphers.

140. RULE.—*Multiply as in integers, and from the right point off as many decimal places in the product as there are decimal places in the two factors.*

NOTE.—To multiply a decimal by 10, 100, 1000, etc., remove the decimal point as many places to the right as there are ciphers in the multiplier, annexing ciphers to the multiplicand, if necessary.

EXAMPLES.

141. *1.* Multiply three hundred and forty-four ten-thousandths by twelve thousandths.

2. Multiply one hundred and ninety-two thousandths by four, and nineteen hundredths.

3. What is sixteen hundredths of six hundred and thirty-two millionths?

4. What is five hundredths of $864.32? Of 3645.75 francs?

5. What is .058$\frac{1}{3}$ of 784.65? Of 943.25?

6. What is $.99 \times 1.106 \times .25$? $4.105 \times .625 \times .512$?

Multiply	Multiply
7. 8.716 by .39; by .047.	*12.* 17.28 by .016$\frac{2}{3}$; by 2.55$\frac{1}{6}$.
8. .00865 by .625; by 97.75.	*13.* 64.32$\frac{1}{2}$ by 1.44$\frac{2}{3}$; by .06$\frac{1}{4}$.
9. .00128 by 8756.8; by 7.865.	*14.* 86.75 by 1.33$\frac{1}{3}$; by 5.76$\frac{2}{3}$.
10. 387.25 by .0147$\frac{1}{3}$; by .087$\frac{5}{6}$.	*15.* 5.78 by .0885; by .66$\frac{2}{3}$.
11. 58.625 by .488$\frac{2}{3}$; by .375.	*16.* 237.5 by .345$\frac{1}{3}$; by 4.468$\frac{1}{8}$.

17. Of 1728, what is .75 ? .33⅓ ? .25 ? .125 ? .20 ? .625 ?

18. Multiply (2.108 + .0074) by (12.684 — .465).

19. Multiply .01837 by 1000 ; .00145 by 100000 ; .6874 by 100 ; 5.375 by 10 ; 17.056 by 10000. What is the sum of the products ?

20. What is the square of .0364 ? Of 20.75 ? Of 45.25 ?

21. What is the cube of 8.045 ? Of .0875 ? Of 67.375 ?

DIVISION.

142. Ex. Divide .01728 by 1.44.

OPERATION.

```
1.44 ) .01728 ( .012
         144
         ---
         288
         288
         ---
           0
```

ANALYSIS.—Dividing as in integers, without reference to the decimal points and prefixed ciphers, the quotient is 12. Since the dividend is the product of the divisor and quotient, it must contain as many decimal places as both of them.

Hence the number of decimal places in the quotient must equal the number in the dividend less the number in the divisor.

If, as in this example, the number of figures in the quotient is less than the number of decimal places to be pointed off, supply the deficiency by prefixing ciphers.

143. RULE.—*Divide as in integers, and point off from the right of the quotient as many decimal places as the number in the dividend exceed those in the divisor.*

NOTES.—1. If the divisor contains more decimal places than the dividend, before dividing make them equal by annexing ciphers to the dividend. If necessary to continue the division, more ciphers may be added.

2. If, after dividing all the figures of the dividend, there is a remainder, the division may be continued by annexing ciphers (**120**). The ciphers thus annexed must be regarded as decimal places of the dividend.

3. To divide a decimal by 10, 100, 1000, etc., remove the decimal point as many places to the left as there are ciphers in the divisor, prefixing ciphers to the dividend, if necessary.

EXAMPLES.

144. *1.* Divide three thousand four hundred and fifty-six hundred-thousandths by seventy-two hundredths.

2. Divide six, and twenty-five hundredths by twenty-five thousandths.

Divide

3. 35.88 by .345; by 4.16.
4. .89958 by .47; by .319.
5. 12.6 by 14.4; by .125.
6. 96.3 by .20; by .25.
7. 5.27 by 1.24; by .85.
8. .0648 by .00425; by .0288.
9. .31752 by .648; by .00384.
10. .1898 by .33⅓; by .0048⅜.
11. 85.2451 by 4.56⅝; by 8.27½.
12. 45.367 by .016⅜; by 1.080⅛.

13. Divide 17.28 by .20; by .25; by .33⅓; by .125; by .66⅔.
14. 321 is .178⅓ of what number?
15. 186 is five hundredths of what number?
16. What must 37.375 be multiplied by to produce 448.5?
17. What must 631.25 be divided by to produce 250?
18. Divide 176.824 by 100; 876.35 by 1000; 17380.5 by 10000; 2886.57 by 10; 375 by 1000000. Find the sum of the quotients.
19. $12.52 is how many hundredths of $375.60?
20. $273.60 is how many thousandths of $1728?

REVIEW EXAMPLES.

145. 1. Add 16 hundredths, 137 millionths, 48 ten-thousandths, and 2016 ten-millionths.

2. Add 16.07, 240.127⅜, 6.04¼, 27.1234.
3. Reduce $\frac{13}{16}$ to a decimal.
4. Reduce .083⅓ to a fraction.
5. From 175 take 16.083⅓.
6. From 375.16⅔ take 198.888⅜.
7. Change .8375 to a fraction.
8. Multiply 117.084 by 7.37½.
9. Divide 43.75 by .0125.
10. Divide .06⅜ by 1.66⅔.
11. 1.75 is $\frac{7}{8}$ of what number?
12. What is $\frac{5}{8}$ of $175.75?
13. What is .33 of 187.5?
14. What is .33⅓ times 1728?
15. $3.75 is how many hundredths of $75?
16. $86.40 is how many hundredths of $2592?
17. 16.56 is .05 of what number?
18. What will 17280 bricks cost at $3.25 per M.?
19. If 278 barrels of pork cost $4378.50, what is the cost of 100 barrels?

20. What cost 12456 feet of plank at $8.75 per M.?

21. What is the value of 5 bbls. sugar, containing 312, 304, 301, 305, 304 pounds respectively, at $9\frac{3}{8}$ cents per pound?

22. A miller wishes to purchase an equal quantity of wheat, corn, and rye; he pays for wheat $2.22\frac{1}{2}$ a bushel; for corn, $98\frac{1}{2}$ cents a bushel; and for rye $1.16\frac{5}{8}$ a bushel. How many bushels of each can he buy for $92776.50?

23. Bought 280 cords of hard wood, at $6.75, and 790 cords of soft wood, at $3.62\frac{1}{2}$ per cord. Also, 750 bushels of corn, at $62\frac{1}{4}$ cents, and 925 bushels of oats, at $37\frac{1}{2}$ cents per bushel. What was paid for the whole, and what was the average price of wood per cord, and of grain per bushel?

24. Bought on contract 350 reams of foolscap paper, at $3.83\frac{1}{2}$ per ream, $45\frac{1}{2}$ reams of which were returned as unsuitable, and 275 reams of letter, at $2.67\frac{1}{2}$ per ream, $37\frac{1}{2}$ reams of which were rejected. How much was paid for the remainder?

25. A merchant paid for merchandise during the year $137618.75, and sold merchandise to the amount of $146347.87. What was the gain, if the net market value of the merchandise remaining unsold was $24378?

26. A quartermaster has $8345 on hand, and receives $4379.62 from each of six sales of property; he turns over to quartermaster A $2875.28, and pays $120 for corn. Upon being relieved from duty, he turns over to quartermaster B one-third of the residue, and divides the remainder equally among three others, C, D, and E. What was paid over to each?

27. Merchandise on hand, Jan. 1, 1879, $46312.85; merchandise sold during the year, $317829.32; merchandise purchased in the same time, $301449.72; merchandise on hand, Dec. 31, 1879, $61378.12. What was the net gain or loss?

28. A farmer sold land for $22.50 an acre, as follows: to A, $98\frac{3}{8}$ acres; to B, $\frac{3}{8}$ of the number sold to A; and to C, $\frac{1}{2}$ the number sold to A and B both. How much land was sold, how much did B and C each receive, and what was the amount realized?

29. What are the prime factors of 2791?

30. At $28.75 per thousand, how many feet of lumber should be given for 2816 pounds of sugar at $7\frac{3}{16}$ cts. per pound?

31. Mr. A offered to sell his horse for $\frac{3}{25}$ more than it cost him, but afterward sold it for $504, which was $\frac{1}{10}$ less than his first asking price. How much did his horse cost him?

32. In England, during the year 1875, there were 147,730,313 tons of bituminous coal produced, 535,845 persons employed, and 1244 lives lost. How many tons of coal were produced to each person employed, how many tons to each life lost, and how many persons were employed per life lost?

33. In the anthracite coal mines of Pennsylvania, during the year 1875, there were 22,000,000 tons of coal produced, 69,589 persons employed, and 238 lives lost. How many tons of coal were produced to each employé, how many to each life lost, and how many persons were employed to each life lost?

34. In the Lehigh district of Pennsylvania, in 1878, there were 3,956,588 tons of coal produced, and 51,492 kegs of powder used. How many tons of coal were produced per pound of powder used, each keg containing 25 pounds?

35. A man bequeaths $\frac{1}{3}$ of his property to his wife, $\frac{1}{4}$ to his son, $\frac{1}{6}$ to his daughter, and the remainder, which is $36375, to charitable institutions. What is the amount bequeathed to each, and the total amount?

36. If a person traveling $3\frac{1}{3}$ miles per hour completes a journey in $16\frac{1}{2}$ hours, what time would it require if he traveled $4\frac{1}{8}$ miles per hour?

37. If I purchase two building lots for $3750 each, and sell one for $\frac{1}{3}$ more than it cost, and the other for $.33\frac{1}{3}$ less, what is the gain or loss on the two lots?

38. A speculator sells two farms for $6000 each; how much does he gain or lose, if he sells one for .20 more than it cost, and the other for $\frac{1}{5}$ less than it cost?

39. A gentleman after spending $\frac{1}{3}$ of all his money, and $\frac{3}{4}$ of the remainder, had $177.50 remaining; how much had he at first?

40. A merchant bought 100 yards of cloth at 3.62\frac{1}{2}$ per yard, and $87\frac{1}{2}$ yards at 4.12\frac{1}{2}$ per yard. At what average price per yard should he sell the whole, to realize a profit equal to $\frac{1}{5}$ of the cost?

41. If $31\frac{1}{4}$ bushels of corn cost $17.50, how many bushels can be bought for $616?

42. *In 1864 there were 33908 miles of railroad in operation in the United States, and in 1878, 81841 miles. What was the average annual increase of mileage?

* This is exclusive of sidings. Mr. Poor, from whose Manual the above was taken, estimates that there are 19,500 miles of railroad in double, treble, and quadruple tracks, sidings, etc., making the total length of single track equal to 101,341 miles in 1878.

DENOMINATE NUMBERS.

Definitions.

146. A **Denominate Number** is a concrete number (7), and may be either *simple* or *compound.*

147. A **Simple Denominate Number** refers to units of the same name and value; as 7 inches, 4 pounds.

148. A **Compound Denominate Number** refers to units of different names, but of the same nature; as 3 feet 6 inches, 4 pounds 8 ounces.

149. Denominate numbers are used to express divisions of time, weights, measures, and moneys of different countries.

150. The scale of integers and decimals is uniform; that of most denominate numbers is varying.

The moneys of the United States, Canada, France, Italy, Spain, Germany, Norway and Sweden, Denmark, Brazil, Japan, and of some other countries, and the metric system of weights and measures, have a uniform decimal scale.

Divisions of Time.

151. The natural divisions of time are the *year* and the *day*, the other divisions being artificial. The year is the time in which the earth makes one revolution around the sun. The day is the time in which the earth makes one revolution on its axis.

152. The **Solar Day** is the interval between two consecutive returns of the sun to the meridian. On account of the varying motion of the earth around the sun, the solar days are of unequal length. For civil purposes in measuring time the average of all the days in the year is taken as the unit.

TABLE.

60 Seconds (*sec.*)	= 1 Minute	*min.*
60 Minutes	= 1 Hour	*hr.*
24 Hours	= 1 Day	*da.*
7 Days	= 1 Week	*wk.*
365 Days, 52 Weeks, 1 day, or 12 Calendar Months	= 1 Common Year	*yr.*
366 Days	= 1 Leap Year	*yr.*
100 Years	= 1 Century	*C.*

NOTE.—In many business transactions the year is regarded as 360 days, or 12 months of 30 days each.

153. The **Calendar Months** with the number of days they contain are as follows:

Season.		Days.
WINTER.	1. January (Jan.)	31.
	2. February (Feb.)	28.
	" in leap year	29.
SPRING.	3. March (Mar.)	31.
	4. April (Apr.)	30.
	5. May	31.
SUMMER.	6. June	30.
	7. July	31.
	8. August (Aug.)	31.
AUTUMN.	9. September (Sep.)	30.
	10. October (Oct.)	31.
	11. November (Nov.)	30.
WINTER.	12. December (Dec.)	31.

154. The **Solar Year** is the time between two consecutive returns of the sun to the vernal equinox. Its exact length is 365 *da.* 5 *hr.* 48 *min.* 50 *sec.* in mean solar time. For civil purposes, the year consists of 365 or 366 days.

In the calendar established by Julius Cæsar, B.C. 46, and thence called the Julian calendar, three successive years were made to consist of 365 days each; and the fourth, of 366 days. According to the Julian calendar, the average length of the year was 365¼ days, thus making an error of 11 min. 10 sec. each year; which in 400 years would amount to 73 hours, or about 3 days. In the sixteenth century, in consequence of the excess of the Julian year above the true solar year, the error in the calendar was 10 days. To correct the calendar, and to prevent any error in the future, Pope Gregory XIII. decreed that 10 days should be omitted in the month of October, 1582, and that all centennial years not divisible by 400 should be common years. Thus, the years 1700, 1800, and 1900, which according to the Julian calendar would be leap years, would according to the reformed calendar be common years. This

calendar is sometimes called the Gregorian calendar. It is now used in all civilized countries except Russia.

The Julian and Gregorian calendars are also designated by the terms Old Style and New Style. In consequence of the years 1700 and 1800 being common years by the Gregorian calendar, the difference between the two styles is now 12 days. Thus, when it is July 4 in Russia, it is July 16 in other countries.

155. RULE FOR LEAP YEARS.—*All years divisible by 4, except centennial years, are leap years.*

All centennial years divisible by 400 are leap years.

LINEAR MEASURE.

156. Linear or **Long Measure** is used in measuring distances, also the length, breadth, and height of bodies, or their linear dimensions.

In measuring length, the yard derived from the standard yard of England is the standard unit, the yards of the United States and England being identical. Theoretically, the yard is equal to $\frac{3600000}{3913393}$ of the length of a pendulum that vibrates seconds in a vacuum, at the level of the sea in the latitude of London; that is, a pendulum that vibrates seconds under the above conditions is 39.1393 inches in length. The standard yard is, in fact, the distance between two points on a brass bar, preserved at Washington, the distance to be taken when the bar is at a temperature of 62° Fahrenheit. This bar was obtained from England in 1827.

TABLE.

		mi.	*fur.*	*rd.*	*yd.*	*ft.*	*in.*
12 Inches (*in.*)	= 1 Foot . . *ft.*	1 =	8 =	320 =	1760 =	5280 =	63360
3 Feet	= 1 Yard . . *yd.*		1 =	40 =	220 =	660 =	7920
5½ Yards	= 1 Rod . . *rd.*			1 =	5½ =	16½ =	198
40 Rods	= 1 Furlong *fur.*				1 =	3 =	36
8 Furlongs	= 1 Mile . . *mi.*					1 =	12

NOTES.—1. The inch is usually divided into halves, quarters, eighths, and sixteenths.

2. The foot and inch are divided by civil engineers and others into tenths, hundredths, thousandths, etc.

3. In measuring cloth, ribbon, and other goods sold by the yard, the yard is divided into halves, quarters, eighths, and sixteenths.

4. At the U. S. Custom Houses the yard is divided into tenths and hundredths.

5. The mile (5280 *ft.*) of the above table is the legal mile of the United States and England, and hence it is sometimes called the statute mile.

157. Other Denominations.—The following denominations are also used:

1 Point	$= \frac{1}{72}$ Inch. } Used by pendulum makers.
1 Line	$= \frac{1}{12}$ Inch. }
1 Size	$= \frac{1}{6}$ Inch. Used by shoemakers.
1 Hand	= 4 Inches. Used in measuring the height of horses.
1 Fathom	= 6 Feet. Used in measuring depths at sea.
1 Cable-length	= 120 Fathoms, or 240 yards.
1 Geographic Mile	= 1.15+ Statute Miles. Used in measuring distances at sea.
1 Knot	= 1 Geo. Mile. Used in determining the speed of vessels.
60 Geo. Miles, or 69.16 Stat. Miles	= 1 Degree of latitude on a meridian, or of longitude on the equator.
360 Degrees	= the Circumference of the Earth.

Surveyors' Linear Measure.

158. Surveyors' Linear Measure is used in measuring land, roads, etc.

The unit of this measure is a chain, 4 rods or 66 feet in length, called Gunter's Chain. It is divided into 100 parts called *links*, each link being 7.92 inches in length.

Table.

100 Links (*l.*) = 1 Chain . . . *ch.*
80 Chains = 1 Mile . . . . *mi.*

mi.	*ch.*	*ft.*	*l.*	*in.*
1 =	80 =	5280 =	8000 =	63360
	1 =	66 =	100 =	792
		.66 =	1 =	7.92

Notes.—1. Links are written decimally as hundredths of a chain.

2. 1 rod = 25 links.

3. Engineers for railroad and other purposes use a chain or tape 100 feet long, the feet being divided into tenths.

Square Measure.

159. Square Measure is used in measuring surfaces.

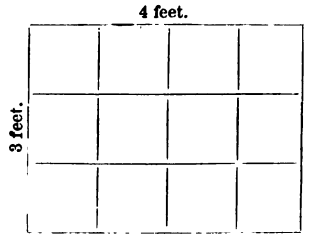
The unit of square measure is a square bounded by lines of some known length. Thus, a square inch is a square whose sides are one inch long; a square foot, a square whose sides are one foot long; etc.

TABLE.

144	Square Inches (*sq. in.*)	= 1 Square Foot . . .	*sq. ft.*
9	Square Feet	= 1 Square Yard . . .	*sq. yd.*
30¼	Square Yards	= 1 Square Rod . . .	*sq. rd.*
160	Square Rods	= 1 Acre	*A.*

NOTE.—1 Rood = 40 *sq. rds.* = ¼ *A.* The rood has practically gone out of use.

160. The **Area** of a surface is an expression for that surface in terms of square units.



In the diagram each small square represents a square foot. Since there are 3 rows, and 4 square feet in each row, there are 3 times 4 square feet, or 12 square feet in the rectangle. Hence, the *area* of any rectangle may be found by multiplying together the numbers denoting its length and breadth, in the same denomination; or, more briefly,

To find the area of a rectangle, multiply its length by its breadth.

SURVEYORS' SQUARE MEASURE.

161. Surveyors' Square Measure is used in measuring land.

TABLE.

10000	Square Links (*sq. l.*)	= 1 Square Chain . . .	*sq. ch.*
10	Square Chains	= 1 Acre	*A.*
640	Acres	= 1 Square Mile . . .	*sq. mi.*

NOTES.—1. 1 Pole or Perch = 1 *sq. rd.* = $\frac{1}{16}$ *sq. ch.* = $\frac{1}{160}$ *A.*

2. The acre is the common unit of land measure.

3. In the vicinity of St. Louis, and in other parts of the Mississippi valley that were settled by the French, the old French *arpent* is still used as the unit of land measure. It contains about $\frac{5}{6}$ of an English acre.

162. U. S. Public Lands are divided by north and south lines run according to the true meridian, and by others crossing

them at right angles, so as to form *townships* of six miles square.

Townships are subdivided into *sections*, containing, as nearly as may be, 640 acres each, or 1 square mile.

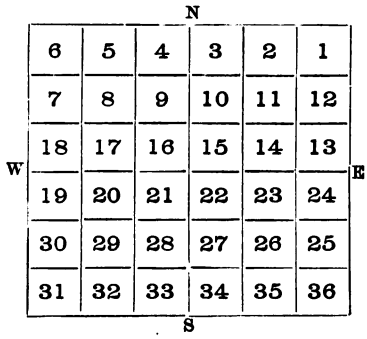
Sections are subdivided into *half-sections*, *quarter-sections*, *half-quarter-sections*, and *quarter-quarter-sections*.

TABLE.

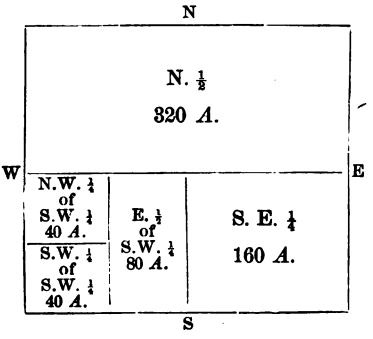
1 Township	= 6 *mi.*	× 6 *mi.*	= 36 *sq. mi.*	=	23040 *A.*
1 Section	= 1 "	× 1 "	= 1 "	=	640 "
1 Half-Section	= 1 "	× $\frac{1}{2}$ "	= $\frac{1}{2}$ "	=	320 "
1 Quarter-Section	= $\frac{1}{2}$ "	× $\frac{1}{2}$ "	= $\frac{1}{4}$ "	=	160 "
1 Half-Quarter-Section	= $\frac{1}{2}$ "	× $\frac{1}{4}$ "	= $\frac{1}{8}$ "	=	80 "
1 Quarter-Quarter-Section	= $\frac{1}{4}$ "	× $\frac{1}{4}$ "	= $\frac{1}{16}$ "	=	40 "

The following diagrams show the method of numbering the sections of a township, as also that of naming the subdivisions of sections.

A TOWNSHIP.



A SECTION.



SOLID OR CUBIC MEASURE.

163. Solid or **Cubic Measure** is used in measuring solids, or bodies, which have length, breadth, and thickness or depth; as boxes, earth, wood, stone, etc.

The unit of cubic measure is a cube, each of whose edges is a unit of some known length. Thus, a cubic inch is a cube, each of whose edges is one inch; a cubic foot is a cube, each of whose edges is one foot; etc.

TABLE.

1728 Cubic Inches (*cu. in.*)	= 1 Cubic Foot	*cu. ft.*
27 Cubic Feet	= 1 Cubic Yard	*cu. yd.*

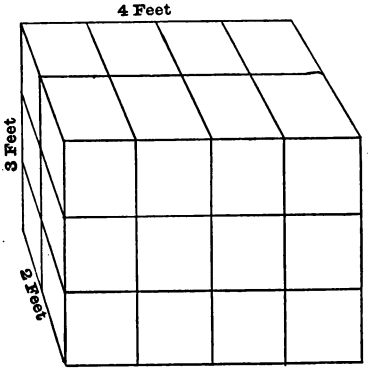
NOTES.—1. 128 cubic feet = 1 cord of wood, or bark. Tanners, in measuring bark, use a measure in which the foot is divided into tenths.

2. The U. S. measurement ton for freight contains 40 cubic feet.

3. The U. S. register tonnage (entire internal cubical capacity) of vessels is expressed in tons of 100 cubic feet each.

164. The **Volume** or **Solid Contents** of a solid is an expression for that solid in terms of cubic or solid units.

The diagram represents a solid 4 feet long, 3 feet broad, and 2 feet thick. Each small cube is a cubic foot. Since the end of the solid contains (3 × 2) 6 square feet of surface, it is evident, if a section 1 foot thick be cut off from this end, it can be divided into 6 cubes, with edges 1 foot in length, and therefore the section will contain 6 cubic feet; and since the whole solid is 4 feet long, and contains 4 like sections, it must contain 4 times 6 cubic feet, or twenty-four cubic feet. Hence the volume of a rectangular solid may be found by multiplying together the numbers expressing its length, breadth, and thickness, in the same denomination; or, more briefly,



To find the volume of a rectangular solid, multiply together its length, breadth, and thickness.

165. **Lumber** is measured by *board measure.* The board foot is 1 *ft.* long, 1 *ft.* wide, and 1 *in.* thick; hence it is $\frac{1}{12}$ of a cubic foot.

Boards, plank, scantling, joists, and sawed timber generally are usually measured by *board* measure; hewn and round timber by *cubic* measure.

166. When lumber is not more than one inch thick, to find the number of feet board measure: *Multiply the length in feet by the width in inches, and divide the product by 12.*

When more than 1 inch thick: *Multiply the length in feet by the width and thickness in inches, and divide the product by 12.*

Liquid Measure.

167. Liquid Measure is used for measuring liquids.

The unit of this measure is the wine gallon, which contains 231 cubic inches.

Table.

		gal.	qt.	pt.	gi.
4 Gills (*gi.*)	= 1 Pint . . . *pt.*	1	= 4	= 8	= 32
2 Pints	= 1 Quart . . . *qt.*		1	= 2	= 8
4 Quarts	= 1 Gallon . . *gal.*			1	= 4

Notes.—1. In estimating the capacity of tanks, cisterns, reservoirs, etc., 1 barrel = 31½ gallons; 1 hogshead = 2 barrels = 63 gallons.

2. In commerce, the barrel and hogshead are not fixed measures, but their capacity is found by gauging, or actual measurement.

3. The imperial gallon of England contains 277.274 cubic inches, and is equivalent to 1.2 U. S. wine gallons.

4. The beer gallon contains 282 cubic inches. It is no longer used in the United States.

Apothecaries' Fluid Measure.

168. Apothecaries' Fluid Measure is used in prescribing and compounding liquid medicines.

The gallon and pint of this measure are the wine gallon and pint.

Table.

		Cong.	O.	℥.	ʒ.	♏.
60 Minims (♏)	= 1 Fluidrachm . ʒ.	1	= 8	= 128	= 1024	= 61440
8 Fluidrachms	= 1 Fluidounce . ℥.		1	= 16	= 128	= 7680
16 Fluidounces	= 1 Pint . . . . *O.*			1	= 8	= 480
8 Pints	= 1 Gallon . . . *Cong.*				1	= 60

Notes.—1. *Cong.* is for the Latin *congius*, gallon; *O.*, for the Latin *octarius*, one-eighth.

2. The symbols precede the numbers to which they refer; thus, *O.* 6 ℥ 10, is 6 pints 10 fluidounces.

Dry Measure.

169. Dry Measure is used in measuring dry articles; as salt, grain, fruits, etc.

The unit of this measure is the Winchester bushel, which contains 2150.42 cubic inches.

TABLE.

		bu.	pk.	qt.	pt.
2 Pints (*pt.*)	= 1 Quart . . . *qt.*	1	= 4	= 32	= 64
8 Quarts	= 1 Peck . . . *pk.*		1	= 8	= 16
4 Pecks	= 1 Bushel . . . *bu.*			1	= 2

NOTES.—1. The half-peck or gallon of this measure contains 268.8 cubic inches.

2. The imperial bushel of England contains 2218.19 cubic inches, and is equal to 1.03 Winchester bushels.

3. Grain, seeds, etc., are usually sold by weight. For table of equivalents see Art. 173.

TROY WEIGHT.

170. Troy Weight is used in weighing gold, silver, coins, and jewels; also in philosophical experiments.

The unit of weight is the Troy pound, which contains 5760 grains. A cubic inch of distilled water weighs 252.458 of these grains, when the height of the barometer is 30 inches, and the temperature of the air and water 62° Fahrenheit.

TABLE.

		lb	oz.	pwt.	gr.
24 Grains (*gr.*)	= 1 Pennyweight *pwt.*	1	= 12	= 240	= 5760
20 Pennyweights	= 1 Ounce . . . *oz.*		1	= 20	= 480
12 Ounces	= 1 Pound . . . *lb.*			1	= 24

NOTE.—The carat, used in weighing diamonds, equals 3.2 Troy grains.

The term carat is also used to denote the fineness of gold, and means $\frac{1}{24}$ part. Thus, gold 18 carats fine contains 18 parts pure gold and 6 parts alloy.

APOTHECARIES' WEIGHT.

171. Apothecaries' Weight is used in prescribing and compounding medicines not liquid.

The pound, ounce, and grain of this weight are the same as those of Troy weight, the division of the ounce being different.

TABLE.

		℔	℥	ʒ	℈	gr.
20 Grains (*gr.*)	= 1 Scruple . . *sc.* or ℈.	1	= 12	= 96	= 288	= 5760
3 Scruples	= 1 Dram . . *dr.* or ʒ.		1	= 8	= 24	= 480
8 Drams	= 1 Ounce . . *oz.* or ℥.			1	= 3	= 60
12 Ounces	= 1 Pound . . *lb.* or ℔.				1	= 20

NOTES.—1. The symbols precede the numbers to which they refer; thus, ℥ 6 ʒ 4, is 6 ounces 4 drams.

2. Drugs and medicines are sold in large quantities by Avoirdupois weight.

AVOIRDUPOIS WEIGHT.

172. Avoirdupois Weight is used in weighing all articles, excepting gold, silver, precious stones, and medicines in small quantities.

The Avoirdupois pound contains 7000 Troy grains.

TABLE.

			T.	*cwt.*	*lb.*	*oz.*
16 Ounces (*oz.*)	= 1 Pound . . . . *lb.*					
100 Pounds	= 1 Hundred-weight, or *cwt.*		1	= 20	= 2000	= 32000
	= 1 Cental . . . . *C.*			1	= 100	= 1600
20 Hundred-weight	= 1 Ton . . . . . *T.*				1	= 16

NOTES.—1. The ounce is divided into halves and quarters.

2. The dram, $\frac{1}{16}$ of an ounce, is now little used, except by silk manufacturers.

3. The Long or Gross ton, formerly used, contained 2240 pounds; the hundred-weight, 112 pounds; and the quarter, 28 pounds.

These weights are still used at the U. S. Custom Houses, in ocean freights, and in freighting and wholesaling coal from the mines.

173. In buying and selling grain, seeds, and other produce, the bushel is regarded as a certain number of pounds. The Boards of Trade of several of our leading cities, and the people generally, use the equivalents given in the following table:*

* These weights are the same as prescribed by the laws of most States, but the laws are not uniform. In inter-state commerce it is necessary to have common units, although they may differ from the units established by law. The laws are generally disregarded where the units prescribed by them differ from those prescribed by custom, or the laws of most of the States. As an instance of this irregularity, the State of New York prescribes 58 pounds as a bushel of corn, but the Boards of Trade and custom generally adopt 56 pounds as a bushel of corn. There can be no doubt but that an appeal to the courts of any one of the States would lead to a decision in accordance with the laws of that State in fixing the weight of a bushel of grain. It is further evident that decisions in State courts of last appeal might be as discordant upon this subject as the laws themselves.

TABLE OF AVOIRDUPOIS POUNDS IN A BUSHEL.

Commodities.	Lbs.	Commodities.	Lbs.	Commodities.	Lbs.
Barley.....	48	Corn, shelled...	56	Peas.....	60
Beans.....	60	Corn in the ear.	70	Rye.....	56
Buckwheat.....	48	Malt.....	34	Timothy Seed..	45
Clover Seed ...	60	Oats.....	32	Wheat.....	60

In the Liverpool, San Francisco, and some other markets, produce is bought and sold by the *cental* of 100 pounds. Railway freight tariffs in the United States on grain, provisions, etc., are reckoned per *cwt.* or cental.

174. The following units are used in commerce:

1 Quintal of Fish	=	100 *lbs.*
1 Barrel of Flour	=	196 *lbs.**
1 Barrel of Pork	=	200 *lbs.*
1 Gallon Refined Petroleum	=	6½ *lbs.*
1 Gallon Crude Petroleum	=	6½ *lbs.*
1 Keg of Nails	=	100 *lbs.*

CIRCULAR MEASURE.

175. Circular or **Angular Measure** is used in measuring angles and arcs of circles. It is employed principally by surveyors in determining directions, by navigators in determining latitude and longitude of places, and by astronomers in making observations.

The unit of this measure is the *degree*, which is $\frac{1}{360}$ of the circumference of any circle.

TABLE.

60 Seconds (″)	= 1 Minute	. . . . .	′.
60 Minutes	= 1 Degree	. . . . .	°.
360 Degrees	= 1 Circle	. . . . .	C.

In order to prevent confusion, to remove the discrepancies which now exist, and to facilitate commerce, it is to be hoped that Congress will enact general laws on the subject, making the equivalents of a bushel uniform, or introducing the cental (or still better the metric) system.

* It is recommended by the leading Boards of Trade that all barrel flour contain 200 pounds, and all sack flour 50, 100, 150, or 200 pounds.

Flour is frequently exported from the United States to Great Britain in sacks of 140 pounds each. The sack of Great Britain usually contains 280 pounds.

NOTES.—1. A quadrant is one-fourth of a circle, or 90°.

2. A sextant is one-sixth of a circle, or 60°.

3. 1 minute of the circumference of the earth is called a nautical, or geographic mile, and is about 1.15 statute or common miles.

COUNTING.

176. The following table is used in counting certain articles:

12 Units	= 1 Dozen	. . .	*doz.*	*g. gr.*	*gr.*	*doz.*	*units.*
12 Dozen	= 1 Gross	. . . .	*gr.*	1 =	12 =	144 =	1728
12 Gross	= 1 Great Gross	.	*g. gr.*		1 =	12 =	144
						1 =	12

PAPER.

177. The following table is used in the paper trade:

24 Sheets	= 1 Quire	. . .	*qr.*	*rm.*	*qr.*	*sheets.*
20 Quires	= 1 Ream	. . .	*rm.*	1 =	20 =	480
2 Reams	= 1 Bundle.				1 =	24
5 Bundles	= 1 Bale.					

UNITED STATES MONEY.

178. United States Money is the legal currency of the United States. It consists of gold coins, silver coins, treasury notes, and national bank notes.

179. Legal Tender.—The term legal tender is applied to money which may be legally offered in the payment of debts.

180. The unit of value is the gold dollar of 25.8 grains.

TABLE.

10 Mills	= 1 Cent	. . . .	*c., ct.*
10 Cents	= 1 Dime	. . . .	*d.*
10 Dimes or 100 Cents	= 1 Dollar	. . . .	$.
10 Dollars	= 1 Eagle	. . . .	E.

NOTES.—1. In business operations, dollars and cents are principally used. Eagles and dimes are used only as the names of coins.

2. In writing U. S. money, the decimal notation is used. *Dollars* are written at the left of the separatrix and form the integral part. *Cents* are written as *hundredths* of a dollar, and occupy the first two places at the right of the separatrix. *Mills* are written as *thousandths* of a dollar, and occupy the third decimal place.

Usually, in the final results of business operations, if the *mills* are more than *five*, they are regarded as an additional cent; if less than *five*, they are rejected.

3. In checks, notes, drafts, etc., *cents* are usually written as hundredths of a dollar in the form of a *fraction*. Thus, *six dollars and twenty-five cents* may be written, $\$6\frac{25}{100}$.

181. The legal coins of the United States are as follows:

GOLD.	Weight in grains.
1 dollar piece,	25.8
2½ dollar piece, or Quarter-eagle,	64.5
3 dollar piece,	77.4
5 dollar piece, or Half-eagle,	129.
10 dollar piece, or Eagle,	258.
20 dollar piece, or Double-eagle,	516.

SILVER.	Weight.
Standard dollar,	412½ grains.
Half dollar, or 50 cent piece,	12½ grams, or 192.9 grains.
Quarter Dollar, or 25 cent piece,	6¼ grams, or 96.45 grains.
Dime, or 10 cent piece,	2½ grams, or 38.58 grains.

COPPER AND NICKEL.	
5 cent piece,	5 grams, or 77.16 grains.
3 cent piece,	30 grains.
1 cent piece,	48 grains.

The Mill is not coined.

182. The **Trade Dollar** contains 420 grains of standard silver (.900 fine). It is not now coined, and is not a *legal tender*. It was originally coined for the purposes of trade in China and Japan.

183. The gold and silver coins of the United States contain 9 parts by weight of pure metal and 1 part alloy. The alloy of silver coins is copper; and the alloy of gold coins, copper, or copper and silver. (The silver in no case exceeds $\frac{1}{10}$ of the whole alloy.)

184. **Gold Coins** are a "*legal tender* in all payments at their nominal value when not below the standard weight and limit of tolerance"* provided by law; and, "when reduced in

* "Any gold coin of the United States, if reduced in weight by natural abrasion not more than one-half of one per centum below the standard weight prescribed by law, after a

weight, below said standard and tolerance, are a legal tender at valuation in proportion to their actual weight."

185. Standard Silver Dollars are "a *legal tender* at their nominal value for all debts and dues, public and private, except where otherwise expressly stipulated in the contract." "The Secretary of the Treasury is authorized and directed to purchase * * * silver bullion * * * not less than $2,000,000 worth per month, nor more than $4,000,000 worth per month, and cause the same to be coined monthly, as fast as so purchased, into such dollars." (Act of Feb. 28, 1878, Sec. 1.)

186. Silver Certificates.—Any holder of *standard silver dollars* "may deposit the same with the Treasurer, or any Assistant Treasurer of the United States, in sums not less than $10, and receive therefor certificates of not less than $10, each corresponding with the denominations of United States notes" (**189**). These certificates are "receivable for customs, taxes, and all public dues, and when so received may be reissued." (Act of Feb. 28, 1878, Sec. 4.)

187. Subsidiary Coins.—"The present (1880) silver coins of the United States of smaller denominations than $1" are "a legal tender in all sums not exceeding $10, in full payment of all dues, public and private." (Acts of 1st session, 46th Congress, Chap. XII, Sec. 3.)

"The holder of any of the silver coins of the United States of smaller denominations than $1 may, on presentation of the same in sums of $20, or any multiple thereof, at the office of the Treasurer or any Assistant Treasurer of the United States, receive therefor lawful money of the United States." (Acts of 1st session, 46th Congress, Chap. XII, Sec. 1.)

188. Minor Coins.—The 5 and 3 cent pieces contain $\frac{3}{4}$ copper and $\frac{1}{4}$ nickel. The 1 cent piece contains 95 per cent. copper and 5 per cent. tin and zinc. These coins are "a legal tender, at their nominal value, for any amount not exceeding twenty-five cents in any one payment."

189. United States Notes ("Greenbacks") are "a legal tender for all debts, public and private, except duties on imports

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circulation of twenty years, as shown by its date of coinage, and at a ratable proportion for any period less than twenty years, is received at its nominal value by the United States treasury and its offices." The "Coinage Act of 1873" allows a deviation from the standard weight of $\frac{1}{2}$ of a grain, or less, in the manufacture of the dollar piece.

and interest on the public debt." Since Jan. 1, 1879, they have been redeemable "in coin * * * on their presentation for redemption at the office of the Assistant Treasurer of the United States in the City of New York, in sums of not less than $50." They represent the values of $1, $2, $5, $10, $20, $50, $100, $500, $1000, $5000, and $10,000. The Act of May 31, 1878, fixed their value at $346,681,016, and forbade their further contraction.

190. National Bank Notes are not a *legal tender;* but, since they are "secured by bonds of the United States deposited with the U. S. Treasurer at Washington," and are redeemed in lawful money by the national banks and the Treasurer of the United States, they are usually accepted in the payment of debts in any part of the United States. They are "receivable in all parts of the United States in payment of all taxes and excises and all other dues to the United States except duties on imports, and also for salaries and other debts and demands owing by the United States to individuals, corporations, and associations within the United States except interest on the public debt."

They represent the values of $1, $2, $5, $10, $20, $50, $100, $500, and $1000. Since Jan. 1, 1879, no notes of the denomination of $1 and $2 have been issued to national banks (R. S. 5175). Since the act of Jan. 14, 1875, the volume of national bank notes has been unlimited. Nov. 1, 1879, their total circulation, including gold banks, was $337,181,418.

ENGLISH MONEY.

191. English or Sterling Money is the legal currency of Great Britain.

TABLE.

				Value in U. S. money.
4 Farthings	=	1 Penny . . .	*d.* . . .	$ .02+
12 Pence	=	1 Shilling . . .	*s.* . . .	.243+
20 Shillings	=	1 Pound, or 1 Sovereign . .	£. . . .	4.8665

NOTES.—1. 1 Crown = 5 shillings, or ¼ of a pound ($1.216+).

2. 1 Guinea = 21 shillings ($5.11). It is not now coined.

3. The gold coins of Great Britain are 22 carats ($\frac{11}{12}$), or .916⅔ fine. (The old carat system (170, note) is generally abandoned except for jewelry. 1 carat = .041⅔.)

4. The silver coins of Great Britain are .925 ($\frac{37}{40}$) fine.

192. FOREIGN MONEYS OF ACCOUNT AND THEIR VALUES IN UNITED STATES MONEY.

Country.	Monetary Unit.	Standard.	Value in U. S. Money.
Austria	Florin of 100 kreutzers.	Silver	.40,7
Belgium	[a]Franc of 100 centimes.	Gold and silver	.19,3
Bolivia	[b]Boliviano, 100 centavos.	Silver	.82,3
Brazil	Milreis of 1000 reis	Gold	.54,6
British America	Dollar of 100 cents	Gold	$1.00
Chili	Peso of 100 centavos	Gold and silver	.91,2
Cuba	Peso of 100 centavos	Gold and silver	.93,2
Denmark	[c]Crown of 100 öre	Gold	.26,8
Ecuador	[b]Peso of 100 centavos	Silver	.82,3
Egypt	Piaster of 40 paras	Gold	.04,9
France	[a]Franc of 100 centimes.	Gold and silver	.19,3
Great Britain	Pound sterling	Gold	4.86.6½
Greece	[a]Drachma of 100 lepta	Gold and silver	.19,3
German Empire	Mark of 100 pfennige	Gold	.23,8
India	Rupee of 16 annas [d]	Silver	.39
Italy	[a]Lira of 100 centesimi	Gold and silver	.19,3
Japan	Yen of 100 sen	Silver	.88,8
Liberia	Dollar of 100 cents	Gold	1.00
Mexico	Dollar of 100 centavos.	Silver	.89,4
Netherlands	Florin of 100 cents	Gold and silver	.40,2
Norway	[c]Crown of 100 öre	Gold	.26,8
Peru	[b]Sol of 100 centavos	Silver	.82,3
Portugal	Milreis of 1000 reis	Gold	1.08
Russia	Rouble of 100 copecks.	Silver	.65,8
Sandwich Islands	Dollar of 100 cents	Gold	1.00
Spain	[a]Peseta of 100 centimes.	Gold and silver	.19,3
Sweden	[c]Crown of 100 öre	Gold	.26,8
Switzerland	[a]Franc of 100 centimes.	Gold and silver	.19,3
Tripoli	Mahbub of 20 piasters.	Silver	.74,3
Turkey	Piaster of 40 paras	Gold	.04,4
U. S. of Colombia	[b]Peso of 100 centavos	Silver	.82,3
Venezuela	[a]Bolivar	Gold and silver	.19,3

The above rates, proclaimed by the Secretary of the Treasury, Jan. 1, 1881, are used in estimating, for Custom-House purposes, the values of all foreign merchandise made out in any of said currencies.

([a]) The *franc* of France, Belgium, and Switzerland, the *peseta* of Spain, the *drachma* of Greece, the *lira* of Italy, and the *bolivar* of Venezuela have the same value.

([b]) The *peso* of Ecuador, and United States of Colombia, the *boliviano* of Bolivia, and the *sol* of Peru have the same value.

([c]) The *crowns* of Norway, Sweden, and Denmark have the same value.

([d]) The *anna* contains 12 *pies*.

REDUCTION.

193. Reduction of Denominate Numbers is the changing their denomination without changing their value.

194. To reduce denominate numbers from higher to lower denominations.

Ex. How many pence in £8 16*s.* 7*d.*?

OPERATION.

£	*s.*	*d.*
8	16	7
	20	
	160*s.*	
	16*s.*	
	176*s.*	
	12	
	2112*d.*	
	7*d.*	
	2119*d.*	

ANALYSIS.—Since there are twenty shillings in 1 pound, in 8 pounds there are 8 times 20 shillings, or 160 shillings. (For convenience multiply by 20 as an abstract number.) 160 shillings plus 16 shillings equal 176 shillings. Since there are 12 pence in 1 shilling, in 176 shillings there are 176 times 12 pence, or 2112 pence. 2112 pence plus 7 pence equal 2119 pence. When possible, add mentally the number of the lower denomination to the product.

195. RULE.—*Multiply the number of the highest denomination given by the number of the next lower denomination required to make 1 of this higher, and to the product add the given number, if any, of such lower denomination.*

Treat this result, and the successive results obtained, in like manner until the number is reduced to the required denomination.

EXAMPLES.

196. Reduce:

1. £9 13*s.* 10*d.* to pence.
2. 6 *gal.* 3 *qt.* 1 *pt.* to gills.
3. £112 18*s.* 5*d.* to farthings.
4. 6 *T.* 12 *cwt.* 65 *lb.* to pounds.
5. The year 1896 to hours.
6. The year 1881 to minutes.
7. £245 15*s.* 3 *far.* to farthings.
8. 48 *bu.* 3 *pk.* 6 *qt.* to quarts.
9. The year 1900 to hours.
10. 18 *lb.* 8 *oz.* to pennyweights.
11. 5 *mi.* 36 *rd.* 11 *ft.* to feet.
12. 456 miles to feet.
13. 16½ hands to inches.
14. 3 *mi.* 46 *ch.* 75 *l.* to links.
15. 7 *mi.* 55 *ch.* to rods.
16. 29 *sq. rd.* to square feet.
17. 97 *sq. rd.* to square yards.
18. 5 *sq. mi.* to acres.
19. 5 miles square to acres.
20. 16 cords 112 *cu. ft.* to *cu. ft.*

21. How many cubic feet in a vessel whose measurement is 2135 tons?

22. How many pounds in 16 *T.* 3 *qr.* 18 *lb.* (Long Ton Table)?

23. How many quarts in 3 *bbl.* 24 *gal.* cider?

24. How many pounds in 2375 bushels corn?

25. At 1 cent each, what is the value of 20 great gross pens?

26. How many days from Jan. 1, 1888, to Jan. 1, 1906?

27. How many days in 8 *m.* 26 *da.*?

197. To reduce denominate numbers from lower to higher denominations.

Ex. Reduce 2119 pence to higher denominations.

OPERATION.

12) 2119*d.*

20) 176*s.* + 7*d.*

£8 + 16*s.*

2119*d.* = £8 16*s.* 7*d.*

ANALYSIS.— Since there are 12 pence in 1 shilling, in 2119 pence there are as many shillings as 12 pence are contained times in 2119 pence, or 176 shillings, and 7 pence remaining. Since there are 20 shillings in 1 pound, in 176 shillings there are as many pounds as 20 shillings are contained times in 176 shillings, or 8 pounds, and 16 shillings remaining. Therefore, 2119*d.* = £8 16*s.* 7*d.*

198. RULE.—*Divide the given number by the number of that denomination required to make 1 of the next higher, reserving the remainder, if any, as part of the answer.*

Treat the quotient, and the successive quotients obtained, in like manner until the number is reduced to the required denomination. The last quotient and the several remainders will form the answer.

EXAMPLES.

199. Reduce

1. 8475*d.* to pounds.

2. 9683 *cu. ft.* to cords.

3. 7534 *pts.* to bushels.

4. 9817 *pts.* to barrels.

5. 5280 *ft.* to miles.

6. 7633 *sq. yds.* to *sq. rds.*

7. 8437 days to *com. yrs.*

8. 6375 *hrs.* to weeks.

9. 9537 *sec.* to hours.

10. 6239 *in.* to rods.

11. 13387*d.* to pounds.

12. 10224 *ft.* to fathoms.

13. 60427 *l.* to chains.

14. 16338 *ft.* to chains.

15. 5384 rods to chains.

16. 6375 *l.* to rods.

17. 5316 *sq. rds.* to acres.

18. 49380 *sq. l.* to acres.

19. 38425 *sq. ch.* to *sq. mi.*

20. 7685 poles to acres.

21. What is the cost of 465 yards of cloth at $9\frac{1}{2}$ pence per yard?

22. What is the value of 49375 pounds of corn at \$0.64 per bushel?

23. What is the value of 27425 pounds of corn at \$0.95 per cental?

24. Required the value of 18643 pounds of oats at 75 *cts.* per bushel.

25. Find the cost of 17387 pounds of oats at \$1.88 per cental.

26. The report of a cannon is heard $4\frac{3}{4}$ seconds after the flash is seen; what is the distance of the cannon, if sound moves 1090 feet per second?

27. What cost 21370 pounds of straw at \$8 per ton?

28. Required the cost of 875 pounds of feed at \$1.15 per *cwt.*

29. In 327 days, how many months of 30 days each?

30. What is the freight of 39445 pounds of merchandise at 64*s.* per ton of 2240 pounds?

REDUCTION OF DENOMINATE FRACTIONS.

200. A Denominate Fraction is a fraction whose integral unit is a denominate number.

The principles, analyses, and rules of denominate fractions are essentially the same as those of denominate integers; therefore, no special rules are necessary for their reduction.

A sufficient number of illustrative examples are given to fully explain the different cases that may arise.

201. To reduce denominate fractions from higher to lower denominations.

Ex. Reduce $\frac{7}{16}$ of a £ to pence.

OPERATIONS.

$$\frac{7}{16} \times \frac{20}{1} = \frac{35}{4}s.$$

$$\frac{35}{4} \times \frac{12}{1} = 105d.$$

Or, $$\frac{7}{16} \times \frac{20}{1} \times \frac{12}{1} = 105d.$$

ANALYSIS.—Since there are 20 shillings in £1, in $\frac{7}{16}$ (.4375) of a £ there are $\frac{7}{16}$ (.4375) of 20 shillings, or $\frac{35}{4}$ (8.75) shillings. Since there are 12 pence in 1 shilling, in $\frac{35}{4}$ (8.75) shillings there are $\frac{35}{4}$ (8.75) times 12 pence, or 105 pence. Or, multiply the given fraction by the numbers of the scale required to reduce its denomination to the required denomination.

Ex. Reduce .4375 of a £ to pence.

OPERATIONS.

$.4375 \times 20 = 8.75s.$
$8.75 \times 12 = 105d.$

Or,

£.4375
20
8.7500*s.*
12
105.0000*d.*

ANALYSIS.—As in previous example.

Ex. Reduce $\frac{7}{16}$ of a £ to integers of lower denominations, *i.e.* to shillings and pence.

OPERATION.

$\frac{7}{16} \times \frac{20}{1} = \frac{35}{4} = 8\frac{3}{4}s.$
$\frac{3}{4} \times \frac{12}{1} = 9d.$
$£\frac{7}{16} = 8s.\ 9d.$

ANALYSIS.—Multiplying by 20, $£\frac{7}{16} = 8\frac{3}{4}$ shillings. Reserve the integral part of the result, and reduce the fractional part to pence. Multiplying by 12, $\frac{3}{4}$ shilling = 9 pence. Hence, $£\frac{7}{16} = 8s.\ 9d.$

Ex. Reduce .4375 of a £ to integers of lower denominations.

OPERATIONS.

$.4375 \times 20 = 8.75s.$
$.75 \times 12 = 9d.$

Or,

£.4375
20
s. 8|.7500
12
d. 9|.0000

ANALYSIS.—Multiplying by 20, £.4375 = 8.75 shillings. Reserve the integral part of the result, and reduce the decimal part to pence. Multiplying by 12, .75 shilling = 9 pence. Hence, £.4375 = 8*s.* 9*d.*

EXAMPLES.

202. *1.* Reduce .625 of a £ to pence.

2. Reduce .875 of a £ to shillings and pence.

3. Reduce $\frac{9}{16}$ of a £ to pence.

4. Reduce $\frac{9}{16}$ of a £ to integers of lower denominations.

5. Change $2.333\frac{1}{3}$ *yrs.* to lower denominations.

6. Change £16.467 to lower denominations.

7. If 1 pound sterling can be bought for $4.87, how many pounds can be bought for $10000 ?

8. Reduce 2.417 *yr.* to lower denominations.

9. A cistern is 16.25 *ft.* long, 9.6 *ft.* wide, and 6.25 *ft.* deep; what is its capacity in *cu. yd.* etc.?

10. A certain sum at a certain rate will in 1 *yr.* produce $60 interest; in what time will the same sum at the same rate produce $15.50 interest?

203. To reduce denominate numbers to fractions (or decimals) of higher denominations.

Ex. Reduce $\frac{3}{5}$ of a penny to the fraction of a £.

OPERATIONS.

$\frac{3}{5} \div 12 = \frac{1}{20}$ *s.*

$\frac{1}{20} \div 20 = \frac{1}{400}$ £.

Or, $\frac{3}{5} \times \frac{1}{\cancel{12}_{4}} \times \frac{1}{20} = \frac{1}{400}$ £.

ANALYSIS.—Divide the given fraction by the numbers of the scale required to reduce pence to pounds.

If the answer is required in the form of a decimal, reduce the resulting fraction to a decimal by Art. 127. $£\frac{1}{400} = £.0025$.

Ex. Reduce .6 of a penny to the decimal of a £.

OPERATION.

12) .6 *d.*

20) .05 *s.*

£.0025

ANALYSIS.—As in previous example.

If the answer is required in the form of a fraction, reduce the resulting decimal to a fraction by Art. 131. $£.0025 = £\frac{1}{400}$.

Ex. Change 9 pence to the fraction of a £.

OPERATIONS.

$\frac{3}{1} \times \frac{1}{\cancel{12}_{4}} \times \frac{1}{20} = \frac{3}{80}$ £.

Or, $£\frac{9}{240} = £\frac{3}{80}$.

ANALYSIS.—For first operation, as in previous example.

Or, since there are 240 pence in £1, 1 penny equals $\frac{1}{240}$ of a £, and 9 pence equal $\frac{9}{240}$, or $\frac{3}{80}$ of a £.

Ex. Reduce 9 pence to the decimal of a £.

OPERATIONS.

12) 9. *d.*

20) .75 *s.*

.0375 £.

ANALYSIS.—As in previous example.

Or, $£\frac{9}{240} = £\frac{3}{80} = £.0375$.

Ex. Reduce 12*s.* 9*d.* to the fraction of a £.

OPERATION.

12*s.* 9*d.* = 153*d.*

£1 = 240*d.*

$\frac{153}{240} = \frac{51}{80}$ £.

ANALYSIS.—12 shillings 9 pence = 153 pence. Since £1 = 240 pence, 1 penny equals $\frac{1}{240}$ of a £. and 153 pence equal $\frac{153}{240}$, or $\frac{51}{80}$ of a £.

Ex. Reduce £18 12*s.* 9*d.* to the decimal of a £.

OPERATION.

12) 9. *d.*
20) 12.75 *s.*
£18.6375

ANALYSIS.—Write the denominations given in a vertical column, the lowest denomination at the top. Since there are 12 pence in 1 shilling, 9 pence are equal to .75 shilling; to which annexing the 12 shillings given, we have 12.75 shillings. Since there 20 shillings in £1, 12.75 shillings are equal to £.6375, to which annexing the £18, we have £18.6375. Hence £18 12*s.* 9*d.* = £18.6375.

EXAMPLES.

204. *1.* Reduce $\frac{5}{8}$ of a penny to the fraction of a pound.

2. Reduce .875 of a shilling to pounds.

3. Change 12 *cwt.* to the decimal of a ton.

4. Reduce 420 grains to the fraction of an ounce Troy.

5. Reduce $\frac{3}{4}$ of a penny to the decimal of a pound.

6. What part of a mile is .165 of a foot?

7. What decimal of a £ are 18*s.* 6*d.* ?

NOTE.—The following method for reducing shillings, pence, and farthings to the decimal of a pound is sufficiently accurate for most business purposes: *Write one-half of the greatest even number of shillings as tenths, and if there be an odd shilling write 5 hundredths; reduce the pence and farthings to farthings, and write their number as thousandths. If the number of farthings is between 12 and 36, add 1 to the thousandths; if between 36 and 48, add 2 to the thousandths.* Thus, £8 17*s.* 8*d.* = £8 + £.85 + £.033 = £8.883.

8. Reduce 116 *cu. ft.* to the decimal of a cord.

9. Reduce £247 14*s.* 9*d.* to pounds.

10. What decimal of an acre are 16 *sq. rds.* ?

11. Reduce 75 feet to the fraction of a mile.

12. Reduce £27 10*s.* 6*d.* to pounds.

13. What is the cost of 22480 pounds of coal at $4.25 per ton (2240 pounds).

14. What is the cost of 16 *tons* 12 *cwt.* of "Nut" coal at $6.80 per ton, and 8 *tons* 16 *cwt.* of "Chestnut" coal at $6.10 per ton?

15. What is the cost of 8364 pounds of oats at $1.65 per cental?

16. What is the cost of 8375 pounds of oats at $0.56 per bushel?

17. If 1 pound is equivalent to 4.87\frac{1}{2}$, what is the value of £1234 16*s.* 9*d.* in U. S. money?

18. Reduce £25 12*s.* 6*d.* to the decimal of a £, and multiply the result by .05.

ADDITION.

205. Denominate numbers are *added, subtracted, multiplied,* and *divided* by the same general methods as are employed for like operations in abstract numbers. The only difference arises from the use of a *varying* scale instead of the uniform scale of 10.

Ex. Add £5 11*s.* 4*d.*, £7 14*s.* 9*d.*, £6 16*s.* 8*d.*, and £7 5*s.* 9*d.*

OPERATION.		
£	*s.*	*d.*
5	11	4
7	14	9
6	16	8
7	5	9
27	8	6

ANALYSIS.—Write the numbers so that like denominations stand in the same column, and begin to add at the right. The sum of the pence is 30*d.* = 2*s.* 6*d.* Write the 6*d.* under the column of pence, and add the 2*s.* to the column of shillings, obtaining for the sum 48*s.* = £2 8*s.* Write the 8*s.* under the column of shillings, and add the £2 to the column of pounds, obtaining for the sum £27; which write under the column of pounds, producing the entire sum, £27 8*s.* 6*d.*

EXAMPLES.

206. *1.* Add £16 5*s.* 4*d.*, £12 8*s.* 9*d.*, £13 14*s.* 8*d.*, £42 0*s.* 7*d.*, and 18*s.* 6*d.*

2. Add 3*T.* 19 *cwt.* 2 *qr.* 16 *lb.*, 4*T.* 13 *cwt.* 3 *qr.* 14 *lb.*, 18*T.* 13 *cwt.* 24 *lb.*, and 42*T.* 8 *cwt.* 1 *qr.* 22 *lb.* (Long Ton Table).

3. Add £163 16*s.* 11*d.*, £52 8*s.* 6*d.*, £3 14*s.* 2*d.*, £84 12*s.* 11*d.*, £106 1*s.* 4*d.*, and £49 13*s.* 8*d.*

4. Add 1 *yr.* 6 *mo.* 10 *da.*, 3 *yr.* 8 *mo.* 24 *da.*, 4 *yr.* 11 *mo.* 16 *da.*, 3 *mo.* 18 *da.*, and 1 *yr.* 8 *mo.* 8 *da.*

5. Add 8 *cd.* 106 *cu. ft.*, 3 *cd.* 85 *cu. ft.*, 2 *cd.* 113 *cu. ft.*, and 5 *cd.* 114 *cu. ft.*

6. Add 16 *hr.* 43 *min.* 48 *sec.*, 3 *hr.* 12 *min.* 40 *sec.*, 1 *hr.* 49 *min.* 13 *sec.*, and 5 *hr.* 19 *sec.*

7. Add 116° 32′ 44″, 8° 28′ 53″, 10° 44′ 12″, and 16° 18′ 13″.

8. Add 12 *ch.* 13 *l.*, 16 *ch.* 92 *l.*, 83 *ch.* 5 *l.*, 4.16 *ch.*, and 5.05 *ch.*

9. Add 1 *lb.* 11 *oz.* 18 *pwt.* 14 *gr.*, 2 *lb.* 8 *oz.* 10 *pwt.*, 4 *lb.* 5 *oz.* 18 *gr.*, and 10 *oz.* 13 *pwt.* 12 *gr.*

10. Add 16 *gal.* 3 *qt.* 1 *pt.*, 45 *gal.* 2 *qt.*, 17 *gal.* 1 *qt.* 1 *pt.*, 4 *gal.* 3 *qt.*, 15 *gal.* 1 *pt.*, and 24 *gal.* 3 *qt.* 1 *pt.*

11. Add £17 16*s.* 8*d.*, £37 13*s.* 5*d.*, £46 7*d.*, £11 5*s.* 10*d.*, £8 4*s.*, £38 19*s.* 3*d.*, and £45 12*s.* 8*d.*

12. Add £175 14*s.* 9*d.*, £37 9*s.* 3*d.*, £5 10*s.* 9*d.*, 17*s.* 3*d.*, £55 17*s.*, £3 6*s.* 9*d.*, £44 18*s.* 5*d.*, £218 15*s.* 6*d.*, and £3 11*s.* 11*d.*

SUBTRACTION.

207. Ex. From £10 6*s.* 4*d.* take £8 15*s.* 3*d.*

OPERATION.		
£	*s.*	*d.*
10	6	4
8	15	3
1	11	1

ANALYSIS.—Write the numbers so that like denominations stand in the same column, and begin to subtract at the right. 3*d.* from 4*d.* leaves 1*d.*, which write under the column of pence. Since 15*s.* cannot be subtracted from 6*s.*, take £1 = 20*s.* from £10, leaving £9, and add it to the 6*s.*, making 26*s.* 15*s.* from 26*s.* leaves 11*s.*, which write under the column of shillings. £8 from £9 leaves £1, which write under the column of pounds. Hence the difference required is £1 11*s.* 1*d.*

EXAMPLES.

208. *1.* From £175 16*s.* 8*d.* take £87 12*s.* 6*d.*

2. From £84 10*s.* 2*d.* take £63 5*s.* 10*d.*

3. From £16 6*s.* 11*d.* take £12 12*s.* 8*d.*

4. From £48 10*s.* 8*d.* take £24 16*s.* 10*d.*

5. From 16 *yr.* 8 *mo.* 10 *da.* subtract 12 *yr.* 5 *mo.* 8 *da.*

6. From 1880 *yr.* 10 *mo.* 16 *da.* take 1876 *yr.* 5 *mo.* 24 *da.*

7. From 1881 *yr.* 4 *mo.* 25 *da.* take 1880 *yr.* 10 *mo.* 15 *da.*

8. From 1882 *yr.* 3 *mo.* 20 *da.* take 1879 *yr.* 8 *mo.* 26 *da.*

9. From 8 *hr.* 16 *min.* 44 *sec.* subtract 6 *hr.* 18 *min.* 40 *sec.*

10. From 105° 43′ 12″ subtract 87° 49′ 16″.

11. From 18 *T.* 16 *cwt.* 3 *qr.* 21 *lb.* take 1 *T.* 2 *cwt.* 2 *qr.* 25 *lb.* (Long Ton Table).

209. To find the interval of time between two dates.

210. There are two methods in common use for finding the time between two dates: 1, by compound subtraction, in which the result is given in years, months, and days, and in which 12 months are considered a year, and 30 days a month; 2, the result is given in days, or in years and days, and the true number of days is taken for each month.

Ex. Find the time in months and days from Apr. 24 to Nov. 10.

OPERATION.	
mo.	*da.*
11	10
4	24
6	16

ANALYSIS.—Represent the months and days by their numbers and find their difference by compound subtraction, writing the later date as the minuend and the earlier as the subtrahend.

In many examples the interval may be found mentally as follows: From Apr. 24 to Oct. 24 are 6 *mo.*; in Oct. there are 6 more days after the 24th (regarding each month as 30 days),

and in November to Nov. 10th inclusive, there are 10 days. Hence the total time between the given dates is 6 *mo.* 16 *da.*

The above methods may be used for finding the exact interval in days by making the necessary corrections. 6 *mo.* 16 *da.* = 196 *da.* From Apr. 24 to Nov. 10, there are 4 months containing 31 *da.* each; hence the true answer is 196 *da.* + 4 *da.*, or 200 *da.*

NOTE.—When the month of February is included, subtract 2 days in a common year, and 1 day in a leap year.

Ex. Find the time from May 18, 1876, to Mar. 2, 1882.

OPERATION.

yr.	*mo.*	*da.*
1882	3	2
1876	5	18
5	9	14

ANALYSIS.—As in preceding example.

Ex. What is the exact number of days from July 20, 1880, to Nov. 10, 1881?

OPERATION.

365 from July 20, 1880, to July 20, 1881.
11 remaining in July.
31 in August.
30 in September.
31 in October.
10 in November.
478 from July 20, 1880, to Nov. 10, 1881.

ANALYSIS.—In finding the interval between two dates the last day is counted, and not the first. Since the time is more than one year, write down 365 days as the number of days from the first date to the same date of the next year. Next write down the number of days in the month of July after the 20th, then the number of days in each of the full calendar months, and finally the number of days in November to Nov. 10 inclusive. The sum of these numbers will be the required time.

EXAMPLES.

211. Find the time by compound subtraction from

1. Jan. 10 to Aug. 28.
2. Mar. 16 to Dec. 4.
3. Feb. 5, 1880, to Oct. 16, 1881.
4. Jan. 27, 1881, to July 4, 1883.
5. May 16, 1882, to Mar. 24, 1884.
6. June 28, 1881, to Apr. 10, 1882.
7. July 30, 1882, to May 12, 1883.
8. Aug. 16, 1883, to Jan. 1, 1885.

Find also the exact number of days between the above dates.

MULTIPLICATION.

212. Ex. Multiply £7 16*s.* 8*d.* by 11.

OPERATION.

£	*s.*	*d.*
7	16	8
		11
86	3	4

ANALYSIS.—11 times 8*d.* are 88*d.* = 7*s.* 4*d.* Write the 4*d.* under the pence, and add the 7*s.* to the product of shillings. 11 times 16*s.* are 176*s.*, plus 7*s.* from the preceding product are 183*s.* = £9 3*s.* Write the 3*s.* under the shillings, and add the £9 to the product of pounds. 11 times £7 are £77, plus £9 from the preceding product are £86, which write under the pounds. Hence the entire product is £86 3*s.* 4*d.*

Ex. Multiply £8 12*s.* 6*d.* by .05.

OPERATION.

12) 6. *d.*
20) 12.5 *s.*
8.625 £
.05
£ .43125
20
s. 8.62500
12
d. 7.50000

ANALYSIS.—Reduce the multiplicand to the decimal of a pound by Art. 203, perform the required multiplication, and reduce the result to shillings and pence by Art. 201.

£8 12*s.* 6*d.* = £8.625
£8.625 × .05 = £.43125
£.43125 = 8*s.* 7.5*d.*

EXAMPLES.

213. *1.* Multiply £17 10*s.* 8*d.* by 9; by 11; by 15.

2. How many cords of wood in 12 loads, each load containing 2 *cd.* 108 *cu. ft.*?

3. What is the cost of 25 *yd.* of silk, at £1 2*s.* 6*d.* per *yd.*?

4. What is .05 of £127 16*s.* 6*d.*? Of £145 15*s.* 9*d.*?

5. What is the weight of 24 silver spoons, each spoon weighing 1 *oz.* 13 *pwt.*?

6. Multiply 1 *hr.* 38 *min.* 22 *sec.* by 15; by 12; by 18.

7. If 15 men perform a certain piece of work in 3 *da.* 16 *hr.* 52 *min.*, how long would it take one man to perform it?

8. Multiply £138 8*s.* 9*d.* by .02½; by .06; by .07.

9. What will 50 *gal.* of wine cost at 8*s.* 3*d.* per gallon?

10. How much grain in 12 bins, each containing 13 *bu.* 3 *pk.* 6 *qt.*?

11. If a man walk 4 *mi.* 3 *fur.* 32 *rd.* in one hour, how far will he walk in 10 hours? In 16 hours?

Division.

214. Ex. If 6 *yds.* of cloth are worth £8 18*s.* 6*d.* what is 1 *yd.* worth?

OPERATION.	£	*s.*	*d.*
6)	8	18	6
	1	9	9

ANALYSIS.—1 *yd.* is worth 1 *sixth* as much as 6 *yd.* $\frac{1}{6}$ of £8 is £1 and £2 remaining. Write the £1 in the quotient, and reduce the £2 to shillings. £2 = 40*s.*, plus 18*s.* in the dividend = 58*s.* $\frac{1}{6}$ of 58*s.* is 9*s.* and 4*s.* remaining. Write the 9*s.* in the quotient, and reduce the 4*s.* to pence. 4*s.* = 48*d.*, plus 6*d.* in the dividend = 54*d.* $\frac{1}{6}$ of 54*d.* is 9*d.*, which write in the quotient. £1 9*s.* 9*d.* is the quotient required.

NOTE.—When the divisor is a denominate number, as in Ex. 2, reduce both divisor and dividend to the same denomination, and divide as in simple numbers.

EXAMPLES.

215. *1.* Divide £13 12*s.* 3*d.* by 11; by 9; by 33.

2. How many yards of muslin at 7*d.* per yard can be bought for £5 12*s.*? For £9 9*s.*? (See note.)

3. Divide 17*s.* 3*d.* by .05; by .09; by .15.

Reduce the dividend to the decimal of a pound, then divide in the usual manner, and reduce the quotient to pounds, shillings, and pence.

4. How many yards of silk at £1 19*s.* 2*d.* per yard can be purchased for £86 3*s.* 4*d.*? (See note.)

5. Divide 85° 18′ 30″ by 15; by 18; by 27.

6. If 48 shares of a certain stock are worth £2013 8*s.* what is the value of 1 share?

7. Divide 322 *A.* 90 *sq. rd.* by 10; by 13; by 16.

8. A pile of wood 4 *ft.* wide and 6 *ft.* high contains 18 *cd.* 72 *cu. ft.*; what is the length of the pile?

9. If 120 spoons weigh 32 *lb.* 9 *oz.* 15 *pwt.*, what does 1 weigh?

10. If 42 *yd.* of cloth cost £20 16*s.* 6*d.*, what is the price of 1 *yd.*? Of 12 *yd.*? Of 20 *yd.*?

Longitude and Time.

216. The whole circle of the earth, or 360°, passes under the sun in 24 hours, and in 1 hour passes $\frac{1}{24}$ of 360°, or 15°; in 1 minute, $\frac{1}{60}$ of 15° (15 × 60′), or 15′; and in 1 second, $\frac{1}{60}$ of 15′ (15 × 60″), or 15″.

217. Comparison of **Longitude** and **Time.**

For a difference of			There is a difference of		
15°	in	Longitude	1 *hr.*	in	Time.
15′	"	"	1 *min.*	"	"
15″	"	"	1 *sec.*	"	"
1°	"	"	4 *min.*	"	"
1′	"	"	4 *sec.*	"	"
1″	"	"	$\frac{1}{15}$ *sec.*	"	"

218. RULE.—*1. The difference in longitude of two places, expressed in ° ′ ″, divided by 15 will produce their difference in time expressed in hours, minutes, and seconds.*

2. The difference in time of two places, expressed in hr. min. sec., multiplied by 15 will produce their difference in longitude expressed in ° ′ ″.

219. TABLE OF LONGITUDES.

Albany	73°	44′	50″ W.	New York	74°	0′	3″ W.
Ann Arbor	80°	43′	W.	New Orleans	90°	2′	30″ W.
Boston	71°	3′	30″ W.	Paris	2°	20′	22″ E.
Berlin	13°	23′	45″ E.	Philadelphia	75°	10′	W.
Calcutta	88°	19′	2″ E.	Rome	12°	27′	14″ E.
Cincinnati	84°	29′	31″ W.	Richmond, Va.	77°	25′	45″ W.
Chicago	87°	37′	45″ W.	San Francisco	122°	26′	45″ W.
Jefferson City, Mo.	92°	8′	W.	St. Paul, Minn.	95°	4′	55″ W.
London	0°	5′	38″ W.	St. Louis, Mo.	90°	15′	15″ W.
Mexico	99°	5′	W.	Washington, D. C.	77°	0′	15″ W.

EXAMPLES.

220. Find the difference in longitude between

1. New York and London.
2. Boston and Paris.
3. Chicago and San Francisco.
4. St. Louis and Calcutta.
5. Philadelphia and Berlin.
6. San Francisco and Calcutta.

Find the difference in time between

7. New York and Greenwich.
8. Chicago and New York.
9. Richmond and Calcutta.
10. Rome and London.
11. Paris and Albany.
12. Calcutta and Jefferson City.

13. The difference in time between New York and Greenwich is 4 *hr.* 56 *min.* $\frac{1}{5}$ *sec.*; what is the difference in longitude? When it is 12 o'clock noon at New York, what is the time at Greenwich?

14. A navigator finds that when it is noon at his place of observation, it is 16 *min.* 34 *sec.* past 10 P.M. by his chronometer, Greenwich time; what is his longitude?

15. When it is 6 o'clock P.M. at Richmond, Va., what is the time at St. Louis, Mo.?

16. If the difference of time between two places is 1 *hr.* 18 *min.* 4 *sec.*, what is the difference of longitude?

17. When it is 20 *min.* past 2 P.M. at Boston, Mass., what o'clock is it at San Francisco?

18. When it is 9 o'clock P.M. in San Francisco, it is 3 *min.* $3\frac{2}{5}$ *sec.* past 11 A.M. in Calcutta; what is the longitude of San Francisco, if the longitude of Calcutta is 88° 19′ 2″ E.?

19. When it is noon in Chicago, it is 5 *min.* $29\frac{1}{2}$ *sec.* of 1 P.M. in New York; what is the longitude of Chicago, the longitude of New York being 74° 3″ W.?

THE METRIC SYSTEM.

221. In the **Metric System**, the *Meter* is the basis of all the weights and measures which it employs.

222. The **Meter** is the unit of length, and is equal to one ten-millionth part of the distance measured on a meridian of the earth from the equator to the pole, and equals about 39.37 inches.

The standard meter is a bar of platinum carefully preserved at Paris. Exact copies of the meter and the other units have been procured by the

* The use of the metric system is (1878) obligatory in Belgium, France, Germany, Greece, Netherlands, Italy, Portugal, Roumania, Spain, and Switzerland; in the Argentine Republic, Brazil, Peru, San Domingo, United States of Colombia, and Uruguay—countries aggregating a population of 181,000,000—while its use is partial or legalized in Austria, Azores, Madeira and Cape de Verde Islands, Central American States, Denmark, Japan, Sweden, Norway, Turkey, Spanish Possessions, Great Britain and the British Possessions, and our own country, aggregating a population of 375,000,000 more. For the year ending June 30, 1877, the value of our imports from countries where the metric system is obligatory amounted to $177,807,469; partially in use, $17,378,735; legalized, $265,211,585; not legalized or in use, only $23,804,140. Of the amount received from countries where its use is legalized, Great Britain and British Possessions furnish $185,667,400. With these countries our present system is partly in harmony, but unfortunately the bulk of our trade with them is made up of articles measured by the bushel and gallon, neither of which standards corresponds to any bushel or gallon of this country. It should be borne in mind that the only legalized system of weights and measures in this country to-day is the metric system, and that this system is the only one we possess in harmony with that of any other country.

several nations, including the United States, that have legalized the system. Comparisons with the standard units are made under certain conditions of temperature and atmospheric pressure.

223. The names of the *higher* denominations, or *multiples*, of the unit are formed by prefixing to the several *units* the Greek numerals, *deka* (10), *hecto* (100), *kilo* (1000), and *myria* (10000); as *dekameter*, 10 meters, *hectometer*, 100 meters, etc.

To assist the memory, observe that the initial letters of the multiples are in alphabetical order; thus, *D*, *H*, *K*, and *M*.

224. The names of the *lower* denominations, or *divisions*, of the unit are formed by prefixing to the several *units* the Latin numerals, *deci* ($\frac{1}{10}$), *centi* ($\frac{1}{100}$), *milli* ($\frac{1}{1000}$); as *decimeter*, $\frac{1}{10}$ meter, *centimeter*, $\frac{1}{100}$ meter, etc.

To assist the memory observe that the following words are derived from the same roots: *dime*, *decimal*, *decimate*, *decennial*, etc.; *cent*, *cental*, *century*, *centennial*, etc.; *mill*, *millennium*, etc.

Linear Measure.

225. Table.

		1 Millimeter.....($\frac{1}{1000}$ of a meter)	=	.03937 *in.*
10 *mm.*	=	1 Centimeter.....($\frac{1}{100}$ of a meter)	=	.3937 *in.*
10 *cm.*	=	1 Decimeter.....($\frac{1}{10}$ of a meter)	=	3.937 *in.*
10 *dm.*	=	1 METER.....(1 meter)	=	39.37 *in.*
10 *m.*	=	1 Dekameter.....(10 meters)	=	32.8 *ft.*
10 *Dm.*	=	1 Hektometer....(100 meters)	=	328.09 *ft.*
10 *Hm.*	=	1 Kilometer.....(1000 meters)	=	.62137 *mi.*

NOTES.—1. The meter, like the yard, is used in measuring cloths, ribbons, laces, short distances, etc.

3 The kilometer is used in measuring long distances, and is about $\frac{5}{8}$ of a mile.

3. The centimeter and millimeter are used by artisans and others in measuring minute lengths. The other denominations are rarely used.

EXAMPLES.

226. *1.* Reduce 875275 meters to kilometers.

ANALYSIS.—Since 1 kilometer equals 1000 meters, in 875275 meters there are as many kilometers as 1000 is contained times in 875275, or 875.275. To divide by 1000 place the point three places to the left (**143**, 3).

2. Reduce 675.318 kilometers to meters.

ANALYSIS.—Since 1 kilometer equals 1000 meters, in 675.318 kilometers

there are 675.318 times 1000, or 675318 meters. To multiply by 1000, place the point three places to the right (**140**, note).

3. Reduce 383.64 meters to centimeters; to kilometers.

4. Reduce 175.16 centimeters to kilometers; to meters.

5. Reduce to meters and find the sum of 876.2 decimeters, 30347 centimeters, 176.48 meters, 8.175 kilometers.

6. A ship sails 5712 kilometers in 48 days; how many kilometers does she sail per day?

7. What is the value of 56.4 meters of silk at $1.75 per meter?

8. 16 pieces of cloth contain 38.5 meters each; 18 pieces contain 39 meters each; and 24 pieces contain 41.2 meters each; how many meters in all?

9. How many meters of ribbon at 27 cents per meter can be purchased for $245.70?

SQUARE MEASURE.

227. The unit of square measure is the *square meter.*

TABLE.

100 Square Centimeters, *sq. cm.* = 1 Square Decimeter = 15.5+ *sq. in.*
100 Square Decimeters, *sq. dm.* = 1 SQUARE METER, *Sq. M.* = 1.196+ *sq. yd.*

NOTES.—1. The square meter is used in measuring flooring, ceilings, etc.; the square decimeter and the square centimeter are used for minute surfaces.

2. Since units of square measure form a scale of hundreds, each denomination must have two places of figures.

228. The unit of **Land Measure** is the *are,* and is equal to a square dekameter (100 square meters), or 119.6 square yards.

TABLE.

1 Centare...(1 square meter) = 1550 *sq. in.*
100 Centares, *ca.* = 1 Are.....(100 square meters) = 119.6 *sq. yd.*
100 Ares, A. = 1 Hectare...(10000 square meters) = 2.471 acres.

NOTE.—The hectare is the ordinary unit for land.

EXAMPLES.

229. *1.* Write 16 *sq. m.,* 8 *sq. dm.,* 24 *sq. cm.,* having the square meter as the unit. *Ans.* 16.0824.

2. Write 83 *sq. m.,* 9 *sq. dm.,* having the *sq. m.* as the unit.

3. In 47 ares how many square meters?

4. In 60.25 hectares how many centares?

5. How many square meters in a building lot 8 *m.* by 32 *m.*?

6. How many building lots, each containing 225 *sq. m.*, can be formed from a field containing 9 hectares?

7. How many hectares in a farm 1.024 *Km.* in width and 1.625 *Km.* in length?

8. What is the cost of a mirror 2.25 *m.* by 1.44 *m.*, at $3.84 per *sq. m.*?

9. How many lots 25 *m.* wide by 60 *m.* deep, or having an equivalent area, can be laid out from 6 hectares?

10. A man bought a piece of land for $6950.50, and sold it for $7603.30, by which transaction he made $6.80 a hectare; how many hectares were there?

11. If the forward wheels of a carriage are 3.5 meters in circumference, and the hind weels 4.8 meters, how many more times will the forward wheels revolve than the hind wheels, in running a distance of 8.4 kilometers?

CUBIC MEASURE.

230. The unit for measuring ordinary solids is the *cubic meter*.

TABLE.

1000 Cu. Millimeters, *cu. mm.*	= 1 Cu. Centimeter	= .061 *cu. in.*
1000 Cu. Centimeters, *cu. cm.*	= 1 Cu. Decimeter	= 61.027 *cu. in.*
1000 Cu. Decimeters, *cu. dm.*	= 1 CU. METER	= 35.317 *cu. ft.* 1.308 *cu. yd.*

NOTES.—1. The cubic meter is used in measuring embankments, excavations, etc.; cubic centimeters and cubic millimeters for minute bodies.

2. Since units of cubic measure form a scale of thousands, each denomination must have three places of figures.

231. The unit of **Wood Measure** is the *ster*, and is equal to a cubic meter, or 35.317 cubic feet.

TABLE.

10 Decisters, *ds.*	= 1 Ster.....(1 Cubic Meter)	= .2759 cord. 35.317 *cu. ft.*
10 Sters, *s.*	= 1 Dekaster, *Ds.*.(10 Cubic Meters)	= 2.759 cords.

EXAMPLES.

232. *1.* Write 29 *cu. m.*, 75 *cu. dm.*, having the cubic meter as the unit. *Ans.* 29.075 *cu. m.*

2. Write 17 *cu. m.*, 218 *cu. dm.*, 27 *cu. cm.*, having the cubic meter as the unit.

3. How many cubic meters in a box 3.5 *m.* by 3.2 *m.* by 2.5 *m.*?

4. Bought 12 sters of wood; having sold 8.7 cubic meters, how much remained?

5. There are 13 blocks of marble, each containing 370.16 *cu. dm.*; how many cubic meters in all?

6. How many cubic meters in an excavation 13.2 *m.* by 18.5 *m.* by 8.4 *m.*?

7. At $1.25 a cubic meter, what will it cost to dig a cellar 6.5 *m.* long, 5.4 *m.* wide, and 2.5 *m.* deep?

8. How many sters of wood in a pile of wood 2.5 *m.* high, 2 *m.* wide, and 16.5 *m.* long? What is the length of a pile of the same height and width containing 216 sters?

DRY AND LIQUID MEASURE.

233. The unit of **Dry** and **Liquid Measure** is the *liter*, which is equal to a cubic decimeter, 1.0567 wine quarts, or .908 dry quart.

TABLE.

		Dry Measure.	Liquid Measure.
	1 Milliliter ...($\frac{1}{1000}$ of a liter) =	.06103 *cu. in.*,	or, .0338 *fl. oz.*
10 *ml.* =	1 Centiliter...($\frac{1}{100}$ of a liter) =	.6103 *cu. in.*,	or, .338 *fl. oz.*
10 *cl.* =	1 Deciliter....($\frac{1}{10}$ of a liter) =	6.1027 *cu. in.*,	or, .845 *gi.*
10 *dl.* =	1 LITER.....(1 liter) =	.908 *qt.*,	or, 1.0567 *qt.*
10 *l.* =	1 Dekaliter...(10 liters) =	9.08 *qt.*,	or, 2.6418 *gal.*
10 *Dl.* =	1 Hectoliter..(100 liters) =	2.8375 *bu.*,	or, 26.418 *gal.*
10 *Hl.* =	1 Kiloliter...(1000 liters) =	28.375 *bu.*,	or, 264.18 *gal.*

NOTES.—1. The liter is commonly used in measuring wine, milk, etc., in moderate quantities. For minute quantities the centiliter and milliliter are employed; and for large quantities the dekaliter.

2. For measuring grain, etc., the hectoliter (= 2.8375 bushels) is commonly used.

3. Instead of the kiloliter and milliliter, it is customary to use their equals, the cubic meter and cubic centimeter.

EXAMPLES.

234. *1.* How many liters in a vessel whose capacity is 1 cubic meter?

2. What is the cost of sixteen liters of milk at 8 cents a liter?

3. How many hectoliters of wheat can be bought for $396 at $5.50 per hectoliter?

4. How many hectoliters of grain can be put in a rectangular bin, 4 *m.* long, 3.5 *m.* wide, and 1.2 *m.* high?

5. How many liters in 63.5 dekaliters? In 83.75 hectoliters?

6. At $1.75 a liter, what is the cost of 85.6 dekaliters of wine?

7. How many hectoliters in 16 cubic meters?

8. How many bags, each holding 1 hectoliter, can be filled from a bin, 1.5 *m.* high, 2.4 *m.* wide, and 5 *m.* long?

9. A cistern 3.5 *m.* by 3.2 *m.*, and 9 *m.* deep, will hold how many dekaliters?

10. A merchant bought 4 hectoliters of nuts at $8.50 per hectoliter, and retailed them at 12 cents a liter; what was his profit?

WEIGHT.

235. The unit of weight is the *gram*, which is equal to the weight of a cubic centimeter of distilled water in a vacuum, at its greatest density (39.2° F.), or 15.432 grains.

TABLE.

		1 Milligram.....($\frac{1}{1000}$ of a gram)	=	.01543 *gr.* Tr.
10 *mg.*	=	1 Centigram ....($\frac{1}{100}$ of a gram)	=	.1543 *gr.* Tr.
10 *cg.*	=	1 Decigram.....($\frac{1}{10}$ of a gram)	=	1.543 *gr.* Tr.
10 *dg.*	=	1 GRAM.....(1 gram)	=	15.432 *gr.* Tr.
10 *g.*	=	1 Dekagram.....(10 grams)	=	.3527 *oz.* Av.
10 *Dg.*	=	1 Hectogram .....(100 grams)	=	3.5274 *oz.* Av.
10 *Hg.*	=	1 Kilogram, or Kilo ...(1000 grams)	=	2.6792 *lb.* Tr. 2.2046 *lb.* Av.
10 *Kg.*	=	1 Myriagram.....(10000 grams)	=	22.046 *lb.* Av.
10 *Mg.*, or 100 Kilos	=	1 Quintal .....(100000 grams)	=	220.46 *lb.* Av.
10 *Q.*, or 1000 Kilos	=	1 Tonneau, or TON ....(1000000 grams)	=	2204.6 *lb.* Av. 1.1023 *T.*

NOTES.—1. The above table is used in computing the weights of all objects from the smallest atom to the largest known body. The gram, kilogram (or kilo), and ton are principally used.

2. The gram is used in weighing letters, gold, silver, precious stones, and medicines.

3. The kilogram, or kilo, like the pound, is used in weighing groceries and coarse articles. It is approximately $2\frac{1}{5}$ pounds Av.

4. The ton is the weight of a cubic meter of water, and is used in weighing very heavy articles, as coal, iron, etc.

5. The pound of Germany, Austria, and Denmark is equal to $\frac{1}{2}$ of a kilogram; the centner, to 100 pounds, or $\frac{1}{2}$ of a quintal.

EXAMPLES.

236. *1.* What is the weight in grams of a cubic meter of water? Of a *cu. dm.* of water?

2. A farmer sells to A 3.716*T.* of hay, to B 4.325*T.*, to C 8775 kilos; how many tons does he sell?

3. The U. S. 50-cent piece weighs 12.5 grams; how many can be coined from a kilogram of standard silver?

4. The U. S. 5-cent piece (copper and nickel) weighs 5 grams; how many 5-cent pieces are equivalent in weight to 12 50-cent pieces?

5. How much alloy must be used in making 1200 U. S. twenty-five-cent pieces. (See Art. 181.)

6. What is the cost of 75.6 kilos of sugar at 18 cents a kilo?

7. How many kilos of lard can be bought for 1248 francs at 1.04 francs per kilo?

8. How many powders, each containing 6 grams, can be made from .372 kilogram?

9. What is the weight of 10 *cu. m.* of ice, it being .93 as heavy as water?

237. SCHEME OF THE WEIGHTS AND MEASURES OF THE METRIC SYSTEM.

Ratios.	Lengths.	Surfaces.	Volumes.	Weights.
1000000....	.....	.....	....	Tonneau.
100000....	... .	.....	.....	Quintal.
10000....	Myriameter..	....	.....	Myriagram.
1000....	Kilometer...	.....	Kiloliter....	Kilogram, or kilo.
100 ...	Hectometer..	Hectare. ..	Hectoliter...	Hectogram.
10....	Dekameter..	.. ..	Dekaliter..	Dekagram.
1 ..	METER.....	ARE.....	LITER.....	GRAM.
$\frac{1}{10}$....	Decimeter...	.....	Deciliter	Decigram.
$\frac{1}{100}$....	Centimeter ..	Centare.....	Centiliter ...	Centigram.
$\frac{1}{1000}$ ...	Millimeter...	.....	Milliliter....	Milligram.

238. Table of Equivalents.

The equivalents here given agree with those that have been established by Act of Congress (July 28, 1866) for use in legal proceedings and in the interpretation of contracts.

1 inch = 2.54 centimeters.....1 centimeter = 0.3937 inch.
1 foot = 3.048 decimeters.....1 decimeter = 0.328 foot.
1 yard = 0.9144 meter.....1 meter = 1.0936 yards = 39.37 in.
1 rod = 0.5029 dekameter.....1 dekameter = 1.9884 rods.
1 mile = 1.6093 kilometers.....1 kilometer = 0.62137 mile.
1 sq. inch = 6.452 sq. centimeters...1 sq. centimeter = 0.155 sq. inch.
1 sq. foot = 9.2903 sq. decimeters....1 sq. decimeter = 0.1076 sq. foot.
1 sq. yard = 0.8361 sq. meter.....1 sq. meter = 1.196 sq. yards.
1 sq. rod = 25.293 sq. meters.....1 are = 3.954 sq. rods = 119.6 sq. yards.
1 acre = 0.4047 hectare.....1 hectare = 2.471 acres.
1 sq. mile = 2.59 sq. kilometers.....1 sq. kilometer = 0.3861 sq. mile.
1 cu. inch = 16.387 cu. centimeters...1 cu. centimeter = 0.061 cu. inch.
1 cu. foot = 28.317 cu. decimeters....1 cu. decimeter = 0.0353 cu. foot.
1 cu. yard = 0.7645 cu. meter.....1 cu. meter = 1.308 cu. yards.
1 cord = 3.624 sters.....1 ster = 0.2759 cord.
1 liquid quart = 0.9463 liter.....1 liter = 1.0567 liquid quarts.
1 gallon = 0.3785 dekaliter.....1 dekaliter = 2.6417 gallons.
1 dry quart = 1.101 liters.....1 liter = 0.908 dry quart.
1 peck = 0.881 dekaliter.....1 dekaliter = 1.135 pecks.
1 bushel = 3.524 dekaliters.....1 hektoliter = 2.8375 bushels.
1 ounce av. = 28.35 grams.....1 gram = 0.03527 ounce av.
1 pound av. = 0.4536 kilogram.....1 kilogram = 2.2046 pounds av.
1 pound av. = 0.9072 German pounds.1 German pound = 1.1023 pounds av.
1 ton (2000 lbs.) = 0.9072 met. ton..1 met. ton = 1.1023 tons = 2204.6 lb. av.
1 grain Troy = 0.0648 gram.....1 gram = 15.432 grains Troy.
1 ounce Troy = 31.1035 grams.....1 gram = 0.03215 ounce Troy.
1 pound Troy = 0.3732 kilogram.....1 kilogram = 2.679 pounds Troy.

EXAMPLES.

239. *1.* In 225 meters how many yards? How many inches?

2. Reduce 6 miles to kilometers; to meters.

3. Reduce 640 acres to hectares; to ares.

4. In 10 kilometers, how many feet? How many miles?

5. In 375.6 kilos, how many pounds?

6. How many German pounds in 225 English or U. S. pounds?

7. What is the weight of the U. S. standard silver dollar in grams? Of the trade dollar?

8. In 5000 U. S. bushels, how many hectoliters? How many dekaliters?

9. In 875 *cu. yd.* how many *cu. m.*?

10. In 1000 *cu. m.* how many *cu. yd.*?

11. Reduce 1728 *gal.* wine to liters; to dekaliters.

12. In 244 *sq. m.* how many *sq. yd.*? How many *sq. ft.*?

13. Reduce 220 *oz.* Av. to grams; to kilograms.

240. Approximate Values.

When no great accuracy is required, we may, for all practical purposes, consider—

1 decimeter	= 4 inches.	1 cu. met. or ster	= 1⅓ cu. yd., or ¼ cord.
1 meter	= 39 inches.	1 liter	= 1 quart.
5 meters	= 1 rod.	1 hectoliter	= 2⅝ bushels.
1 kilometer	= ⅝ mile.	1 gram	= 15½ grains.
1 square meter	= 10¾ square feet.	1 kilogram	= 2⅕ pounds.
1 hectare	= 2½ acres.	1 ton	= 2200 pounds.

Approximate Rules.

241. To reduce avoirdupois ounces to grams:

Multiply by 30, and then deduct one-twentieth (5 per cent.).

Note.—Answer too great by about 5 *g.* for every 1000 *g.* of the result.

242. To reduce avoirdupois pounds to kilograms:

Divide by 2, and then deduct one-tenth.

Note.—Answer too small by about 8 kilos for every 1000 kilos of the result. If $\frac{1}{11}$, instead of $\frac{1}{10}$, be deducted, the answer will be too great by 2 kilos for every 1000 kilos of the result.

243. To reduce avoirdupois pounds to half-kilograms, or German pounds:

Deduct one-tenth.

Note.—The answer by this rule will be too small by about 8 German pounds for every 1000 German pounds of the result. If $\frac{1}{11}$ be deducted, the answer will be too great by 2 German pounds for every 1000 German pounds of the resnlt.

244. To reduce tons (2000 lbs.) to metric tons:

Deduct one-tenth.

Note.—The same relative error as in Art. 242.

245. To reduce yards to meters:

Deduct one-twelfth.

NOTE.—Answer too great by $2\frac{1}{2}$ *m.* for every 1000 *m.* of the result.

246. To reduce square yards to square meters:

Deduct one-sixth.

NOTE.—Answer too small by about 3 *sq. m.* for every 1000 *sq. m.* of the result.

247. To reduce cubic yards to cubic meters:

Divide by 1.3.

NOTE.—Answer too great by about 6 *cu. m.* for every 1000 *cu. m.* of the result.

248. To reduce U. S. gallons to liters:

Multiply by 4, and then subtract one-twentieth (5 per cent.).

NOTE.—Answer too great by about 4 *l.* for every 1000 *l.* of the result.

249. To reduce U. S. bushels to hectoliters:

Divide by 3, and then add one-twentieth (5 per cent.).

NOTE.—Answer too small by about 7 *hl.* for every 1000 *hl.* of the result.

250. To reduce grams to avoirdupois ounces:

Divide by 30, and then add one-twentieth (5 per cent.).

NOTE.—Answer too small by about 8 ounces for every 1000 ounces of the result.

251. To reduce kilograms to avoirdupois pounds:

Multiply by 2, and then add one-tenth.

NOTE.—Answer too small by about 2 *lb.* av. for every 1000 *lb.* av. of the result.

252. To reduce German pounds, or half-kilograms, to avoirdupois pounds:

Add one-tenth.

NOTE.—Same error as in Art. 251.

253. To reduce metric tons to U. S. tons (2000 lbs.):

Add one-tenth.

NOTE.—Answer too small by about 2 U. S. tons for every 1000 tons of the result.

254. To reduce meters to yards:

Add one-twelfth, and 1% of the original number.

NOTE.—Answer will be too small by only $\frac{1}{4}$ *yd.* for every 1000 *yd.* of the result.

This method is used at the New York Custom House and is sufficiently accurate for practical purposes.

If $\frac{1}{11}$ be added, the answer will be too small by about $2\frac{1}{2}$ *yd.* for every 1000 *yd* of the result. If $\frac{1}{10}$ be added, the answer will be too great by about 6 *yd.* for every 1000 *yd.* of the result.

Ex. According to above rule, how many yards in 324 meters?

OPERATION.

324
27
3.24
354.24

(Exact result is 354.33 *yd.* Error only .09 *yd.*)

255. To reduce square meters to square yards:

Add one-fifth.

NOTE.—Answer too great by about 3 *sq. yd.* for every 1000 *sq. yd.* of the result.

256. To reduce cubic meters to cubic yards:

Multiply by 1.3.

NOTE.—Answer too small by about 6 *cu. yd.* for every 1000 *cu. yd.* of the result.

257. To reduce liters to U. S. gallons:

Multiply by 2.11, and then divide by 8.

NOTE.—Answer too small by about 1.7 *gal.* for every 1000 *gal.* of the result.

Ex. In 144 liters, how many U. S. gallons?

OPERATION.

144
144
288
8) 303.84
37.98

(Exact result should be 38.04. Error only .06 *gal.*

258. To reduce hectoliters to U. S. bushels.

Multiply by 3, and then subtract one-twentieth (5 per cent.).

NOTE.—Answer too great by about 4 *bu.* for every 1000 *bu.* of the result.

259. Foreign Weights and Measures.

ARGENTINE CONFEDERATION.

Metric system used in the assessment of duties. Old Spanish weights and measures (see Spain) in common use.

AUSTRIA, (AS GERMANY.)

BELGIUM, (METRIC SYSTEM.)

BOLIVIA.

The metric system is the legal system, but the law has not been rigidly enforced. Old Spanish weights and measures (see Spain) still in use. For coin weight the metric gram is used.

BRAZIL, (METRIC SYSTEM.)

Diamonds are permitted to be sold according to the old Portuguese *outava* (55.34 grains).

Ships' freights are, for the most part, settled according to the English ton (2240 *lb.*).

CANADA, (AS GREAT BRITAIN.)

CAPE OF GOOD HOPE, (AS GREAT BRITAIN.)

CEYLON, (AS GREAT BRITAIN.)

CHILI, (AS BOLIVIA.)

For custom purposes, the metric system is enforced.

CHINA.

1 Tael	=	$1\frac{1}{3}$ *oz.* av.
1 Catty	=	$1\frac{1}{3}$ *lb.* av.
1 Picul	=	$133\frac{1}{3}$ *lb.* av.
1 Chih	=	14.1 inches.
1 Chang	=	11.75 feet.

COLUMBIA, (METRIC SYSTEM.)

DENMARK.

1 Pound ($\frac{1}{2}$ kilogram)	=	1.102 *lb.* av.
1 Centner (100 *lb.*)	=	110.23 *lb.* av.
1 Tönde of grain	=	3.948 U.S. *bu.*
1 Tönde of coal	=	4.825 U.S. *bu.*
1 Fod (Foot)	=	1.03 U.S. *ft.*
1 Viertel	=	2.04 U.S. *gal.*
1 Alen (Ell)	=	.6864 *yd.*

Coinage laws are metric. The introduction of complete metric system is in prospect.

ECUADOR, (METRIC SYSTEM.)

ENGLAND, (SEE GREAT BRITAIN.)

EGYPT, (METRIC SYSTEM.)

FRANCE, (METRIC SYSTEM.)

The old French *aune* = 45 inches is still used to some extent in the silk industries of France and the U. S.

GERMANY.

Metric system with a few changes in subdivisions in general use.

1 Pound ($\frac{1}{2}$ kilogram)	=	1.1023 *lb.* av.
1 Centner (100 pounds)	=	110.23 *lb.* av.
1 Wispel (metric ton)	=	2204.6 *lb.* av.

GREAT BRITAIN.

1 Imp. Gallon	=	1.2 U.S. *gal.*
1 " Bushel	=	1.03 U.S. *bu.*
1 " Quarter	=	8.25 U.S. *bu.*
1 Ale or Beer Gallon	=	1.22 U.S. *gal.*
1 Cental	=	100 *lb.*
1 Quarter of Wheat at London	=	480 *lb.*
1 Quarter of Wheat at Hull and Newcastle.	=	504 *lb.*
1 Quarter of Wheat at Dundee and other places.	=	496 *lb.*

Metric system permitted by law of 1864.

GREECE.

Metric system with the common Grecian names in general use.

In the Ionian Islands the English weights and measures have been legalized since 1829.

HONG KONG, (AS CHINA.)

INDIA.

1 Seer = 16 chattacks.

1 Bombay Maund of	40	seers	= 28 *lb.* av.
1 " "	42	"	= 29.4 "
1 Surat "	40	"	= 31⅓ "
1 " "	42	"	= 39⅕ "
1 " "	44	"	= 41 1/15 "
1 Bengal Factory Maund			= 74⅔ "
1 " Bazaar "			= 82⅕ "
1 Madras Maund			= 25 "
1 Bom'y Candy of	20	maunds	= 560 "
1 Surat " "		"	= 746⅔ "
1 Madras " "		"	= 500 "
1 Travancore " "		"	= 660 "
1 Tola			= 180 *gr.*
1 Guz of Bengal			= 1 Eng. yard.
1 Corge			= 20 units.
1 Corge Pound			= 20 *lb.*

Metric system permissive.

ITALY.

1 Palm = .555 *cu. ft.*

Metric system in general use.

JAPAN.

1 Picul = 133⅓ *lb.* av.

For coinage, in part, the metric unit of weight is used.

JAVA.

1 Amsterdam Pond	=	1.09 *lb.* av.
1 Picul	=	133⅓ "
1 Catty	=	1⅓ "
1 Chang	=	4 yards.

LIBERIA.

British weights and measures generally used.

MEXICO.

Weights and measures are legally the metric, but the metric system is not generally in force, the old Spanish weights and measures (see Spain) being still employed.

NETHERLANDS.

Metric system with a change in names in general use.

1 Last (30 hectoliters) = 85.134 *bu.*

NORWAY AND SWEDEN.

1 Swedish Skalpond	=	0.93⅓ *lb.* av.
1 Swedish Centner	=	93⅓ "
1 Norwegian Pund	=	1.1 "
1 Swedish Fot	=	11.7 inches.
1 Norwegian Fod	=	12.02 "

In Norway the metric system is used to some extent.

In Sweden, the coin weight and the medicinal and apothecary weight are metric. The complete metric system is now permissive, and will be obligatory after 1882.

PERU, (AS BOLIVIA.)

PORTUGAL.

Metric system compulsory since Oct. 1, 1868.

The chief old measures are—

1 Libra	=	1.012 *lb.* av.
1 Almunde of Lisbon	=	4.42 U. S. *gal.*
1 Alquiere	=	.3928 U. S. *bu.*

RUSSIA.

1 Pound	=	0.9 *lb.* av.
1 Pood (63 to a ton)	=	36 "
1 Berkowitz	=	360 "
1 Chetvert	=	5.956 U. S. *bu.*
1 Vedro	=	3.25 U. S. *gal.*
1 Arsheen	=	28 inches.
1 Ship Last	=	2 tons.

Metric system partially in use.

SIAM.

1 Tael = 1⅓ *oz.* av.

Picul, Catty, and Chang, same as Java.

SPAIN, (METRIC SYSTEM.)

In many of the South American States and in Cuba, the old Spanish weights and measures, principally Castilian, are used. They are as follows:

1 Libra	= 1.014 *lb.* av.
1 Arroba (25 Libras)	= 25.36 "
1 Quintal (100 Libras)	= 101.44 "
1 Vara	= .914 *yd.*

SWITZERLAND.

Metric system used with some changes of names and subdivisions. Pure metric system optional.

TURKEY, (METRIC SYSTEM.)

URUGUAY, (AS ARGENTINE CONFEDERATION.)

VENEZUELA, (METRIC SYSTEM.)

REVIEW EXAMPLES.

260. *1.* How many days from Mar. 16 to Oct. 4? From June 30 to Dec. 25?

2. Find the time by compound subtraction from Aug. 23, 1882, to Jan. 15, 1884.

3. How many leap years from 1881 to 1897? From 1795 to 1845? From 1889 to 1909?

4. Reduce 2.375 years to years, months, and days.

5. Suppose a person's income to be $1000 per day, how much is that per minute?

6. How many chains in one mile?

7. In 4376 feet how many chains? How many inches?

8. In 396 rods, how many chains? How many feet?

9. In 37.56 chains, how many feet? How many rods?

10. Children's size 1 of shoemakers' measure is $4\frac{1}{3}$ inches long; what is the length of boys' size 8, youths' size 1, and men's size 10? (Size 1 of the second series is one size longer than size 13 of the first series. See Art. 157.)

11. How many square feet in a rectangular lot, whose breadth is $25\frac{3}{4}$ feet and whose length is $116\frac{1}{2}$ feet?

12. How many square feet in a lot 25 feet front and 100 feet deep?

13. How many acres in a rectangular field, 28.50 chains by 46.38 chains?

14. How many acres in a rectangular piece of land, 224 links by 448 links?

15. How many acres in a square lot whose side is $316\frac{1}{4}$ links? 208.71 feet?

16. How many square yards in a floor, 16 *ft.* 6 *in.* by 12 *ft.* 9 *in.*?

17. How much will it cost to carpet a floor 16 *ft.* by 18 *ft.*, with carpeting $\frac{3}{4}$ *yd.* wide, at \$1.60 per yard?

18. What is the value of a field 320 *rd.* long and 160 *rd.* wide at \$22.50 an acre?

19. A rectangular lot contains 24 acres; what is its width, its length being 1056 feet?

20. How much will it cost to dig a cellar 36 *ft.* long, 30 *ft.* wide, and 6 *ft.* deep, at 30 cents per cubic yard?

21. If a pile of bark is 40 *ft.* long and 4 *ft.* wide, how high must it be to contain 10 cords?

22. How many feet, board measure, in 16 boards each 18 *ft.* long, 10 *in.* wide, and 1 *in.* thick?

23. How many feet, board measure, in 12 planks, each 10 *ft.* long, 12 *in.* wide, and 2 *in.* thick?

24. How many board feet in 225 cubic feet?

25. What is a pile of wood, 19 *ft.* long, 11 *ft.* 5 *in.* high, and 8 *ft.* 7 *in.* broad, worth, at \$5.62$\frac{1}{2}$ per cord?

26. Paid \$222.75 for boards at \$13.50 per M.; how many feet were purchased?

27. What is the value of 27315 *ft.* of lumber at \$12 per M.?

28. How many pills, each containing 5 grains, can be made from 1 *lb.* av. of quinine?

29. In 70 *oz.* Tr., how many *oz.* av.?

30. In 70 *lb.* Tr., how many *lb.* av.?

31. What is the cost of 11 *T.* 12 *cwt.* of "Nut" coal at \$6.95 per ton, and 9 *T.* 16 *cwt.* of "Chestnut" coal at \$6.25 per ton?

32. What is the freight of 16 *T.* 17 *cwt.* 25 *lb.* at \$5 per ton (2240 *lb.*)?

33. What is the cost of 15669 pounds meal at \$1.10 per *cwt.*?

34. What cost 16450 pounds of hay at \$15.50 per ton?

35. In 27318 pounds of corn, how many bushels? What is the value of the same at 48$\frac{3}{4}$ cents per bushel?

36. What is the value of 27318 pounds of corn, at 87.1 cents per cental?

NOTE.—Examples 35 and 36 illustrate the present and the cental systems of buying and selling produce, and show the calculations saved by using the latter.

37. In 7346 pounds of oats, how many bushels?

38. What is the cost of 273$\frac{1}{2}$ *bu.* oats, at 58 *c.* per bushel?

39. What is the value of 281 *lb.* peas at \$1.05 per bushel?

40. What is the value of 291 *lb.* of peas at $1.75 per cental?

41. What is the value of 186 *lb.* of beans at $2.25 per bushel?

42. What is the cost of 192 *lb.* of beans at $3.75 per cental?

43. At what price per bushel is rye at $1.227 per cental? Oats at $1.66 per cental? Barley at $2.126 per cental? Malt at $2.75 per cental?

44. How many bushels in 27316 pounds of wheat? In 24375 pounds of corn? In 16218 pounds of oats? In 21412 pounds of barley? In 17387 pounds of malt?

45. How many bushels in 54 centals of wheat? In 87 centals of corn? In 46 centals of oats? In 53 centals of barley? In 67 centals of malt?

46. How much per cental, is wheat at $1.85\frac{1}{2}$ per bushel? Corn at $76\frac{1}{4}$ cents per bushel? Oats at $48\frac{3}{4}$ cents per bushel? Barley at 87 cents per bushel?

47. How much per bushel is wheat at $1.27 per cental? Corn at $1.323 per cental?

48. How much per cental is timothy seed at $1.75 per bushel? Clover seed at $8.55 per bushel?

49. What is the cost of 561^{23} bushels oats at 43 cents per bushel? Of 411^{14} bushels corn at 46 cents per bushel?

50. A quartermaster purchased 75000 pounds of corn, at $31\frac{1}{4}$ cents per bushel; 32113 pounds of oats, at $32\frac{1}{2}$ cents per bushel; and 79500 pounds of hay, at $22.37\frac{1}{2}$ per ton (2000 pounds). What was the total cost of the purchase?

51. A farmer sold 18360 pounds of corn, at 64 cents per cental; 22450 pounds of oats, at 94 cents per cental; and 36650 pounds of hay, at $1.31 per cental. How much was realized from the sale?

52. How many sheets of paper in 5 reams?

53. When 1 gold dollar was worth $2.85 in currency, what was the value of the legal tender dollar in gold?

54. How many grains of gold and alloy respectively are required for the coinage of 6983 gold dollars?

55. How many Troy ounces of pure silver would be required in the coinage of 2,000,000 standard silver dollars? How much copper?

56. What is the avoirdupois weight of 100000 double-eagles, 25000 eagles, 1000 half-eagles, 4000 quarter-eagles, and 1983 gold dollars?

57. What is the value of an *oz.* Tr. of standard gold, making no allowance for the alloy and coinage? Of an *oz.* av.?

58. What is the value of an *oz.* Tr. of pure gold, making no allowance for the alloy and coinage? Of an *oz.* av.?

59. Feb. 26, 1879, the Nevada Bank of San Francisco sold 100,000 ounces of pure silver to the United States, at $1.08\frac{1}{2}$ per ounce. At this rate, what is the intrinsic gold value of the standard silver dollar?

60. The coinage at the mints of the United States during the fiscal year ending June 30, 1879, was as follows:

GOLD—Double-eagles, $57,234,340; eagles, $1,031,440; half-eagles, $1,442,130; three-dollars, $109,182; quarter-eagles, $1,166,800; dollars, $3,020; total gold, $———.

SILVER—Dollars, $27,227,050; Half-dollars, $225; quarter-dollars, $112.50; dimes, $45; total silver, $——.

MINOR COINAGE—5-cents, $1,175; 3-cents, $984; cents, $95,639; total minor coinage, $———.

How many pieces were coined and what was the total value of the coinage?

61. Add £27 16*s.* 10*d.*, £6 10*s.* 8*d.*, £47 15*s.* 11*d.*, £25 7*s.* 6*d.*, £3 14*s.* 8*d.*, and £23 16*s.* 3*d.*

62. In 47 guineas, how many shillings and pounds?

63. What is the value of 45000 tons of steel rails at 97*s.* 6*d.* per ton? What is the value per ton in U. S. money? Of total in U. S. money?

64. How many yards of cloth at 3*s.* 6*d.* per yard can be bought for £7?

65. Reduce £19 16*s.* 9*d.* to the decimal of a pound.

66. If £1 sterling is worth $4.87, what is the value of £225 18*s.* 6*d.*?

67. From £16 12*s.* 9*d.* deduct .05 of itself.

68. What is the value of 20 *yd.* silk at 10*s.* 6*d.* per yard?

69. If 1 franc is worth $.193, what is the value of $1 in francs?

70. What is the value in U. S. money of 875 Napoleons? (1 Napoleon = 20 francs.)

71. What is the cost of 50 meters silk at 8.25 francs (8 francs 25 centimes) per meter?

72. What is the value in U. S. money of 24000 marks?

73. What is the value in U. S. money of 5,528,364 Brazilian reis? Of 7387 Portuguese milreis?

74. In 8375 pies (money of India), how many annas and rupees?

75. What is the value in dollars of 500 Russian poods of rye at 75 copecks per pood?

76. The gold yen of Japan contains $1\frac{1}{2}$ grams of fine gold and weighs $1\frac{2}{3}$ grams. What is its fineness, and what is its intrinsic value compared with the U. S. gold dollar? How many yens can be coined from 10 grams of Japanese standard gold?

77. The difference in the local time of two places is 3 *hr.* 43 *min.* 12 *sec.*; what is the difference in longitude?

78. When it is 4 *hr.* 40 *min.* A.M. at Chicago, what is the time at Calcutta?

79. How many bushels will a box 10 *ft.* long, 5 *ft.* wide, and 4 *ft.* high contain?

NOTE.—Since a bushel is about $1\frac{1}{4}$ cubic feet, the following approximate rules may be used for all practical purposes:

To reduce cubic feet to bushels: *Deduct one-fifth.*

The result will be too small by about $4\frac{1}{2}$ bushels for every 1000 bushels of the result.

To reduce bushels to cubic feet: *Add one-fourth.*

The result will be too great by about $4\frac{1}{2}$ cubic feet for every 1000 cubic feet of the result.

Solve the above example, both exactly and approximately, and compare the results.

80. How many hectoliters of grain will a box 4 meters long, 3.2 meters wide, and 2.5 meters high contain?

81. How many gallons of water will a cistern hold which is 8 *ft.* long, 7 *ft.* wide, and 10 *ft.* deep?

82. What is the capacity in liters of a cistern 25 meters long, 2.2 meters wide, and 3 meters deep?

83. In 52 meters cassimere, how many yards?

84. The specific duty on Brussels carpet is 44 cents per square yard; what is the duty per square meter?

85. In a pane of glass 24 *in.* by 30 *in.*, how many square decimeters?

86. The duty on pig-iron is \$7 per ton (2240 *lb.*); what is the duty per metric ton or millier?

87. The U. S. custom duty on alcohol is \$2 per gallon; what is the duty per liter?

88. The duty on tallow candles is $2\frac{1}{2}$ cents per pound; what is the duty per kilogram?

PERCENTAGE.

261. Percentage is a term applied to all operations in which 100 is used as the basis of computation.

It is also the name given to any number of hundredths of a number.

262. Per Cent. (%) is an abbreviation of the Latin *per centum*, meaning *on* or *by the hundred.*

Thus, 5% means 5 of every hundred, or 5 hundredths ($\frac{5}{100}$, or .05).

263. Any *per cent.* may be expressed in the form of a *decimal* or *fraction.*

Thus 5 *per cent.* = 5% = 5 hundredths = .05 = $\frac{5}{100} = \frac{1}{20}$. The first two forms are used in the statements of questions; the others in the operations.

264. In percentage, three elements are considered, viz: the *Base,* the *Rate,* and the *Percentage.* Any two being given, the other can be found.

265. The **Percentage** is the result obtained by taking a certain number of hundredths of a number.

266. The **Base** is the number of which a certain number of hundredths are taken.

267. The **Rate** is the number of hundredths, or the number per cent.

Thus, in the statement, 6% of 300 is 18, the Percentage is 18, the Base 300, and 6 per cent. (.06) is the Rate.

268. Applications of Percentage.—The principles of percentage are applied to many of the most common business transactions. Among the most important of these are Trade Discounts, Commission, Insurance, Profit and Loss, Duties, Interest, and Exchange.

269. Ex. What is 5 per cent. of 300?

OPERATION.

300 Base.
.05 Rate.
15.00 Percentage.

ANALYSIS.—5% is equivalent to 5 hundredths ($\frac{5}{100}$, or .05). 5 hundredths of a number may be found by multiplying it by 5 hundredths. For convenience, the multiplication is performed by expressing the 5 hundredths in the form of a decimal. $.05 \times 300 = 15$, the percentage required. Therefore, the Percentage is the product of two factors, the Base and the Rate.

Or, 1% of 300 is 3, and 5% is 5 times 3, or 15.

Ex. 15 is 5 per cent. of what number?

OPERATION.

Rate. Percentage.
.05) 15.00
Base. 300.

ANALYSIS.—In this example there is given the Percentage and Rate, to find the Base. Since the Percentage = the Base × the Rate, the Base = the Percentage ÷ the Rate.

Or, if 15 is 5% of a certain number, 1% is $\frac{1}{5}$ of 15, or 3; and the number, or 100%, is 100 times 3, or 300.

Ex. 15 is what per cent. of 300?

OPERATION.

Base. Percentage. Rate.
300) 15.00 (.05

ANALYSIS.—The Base and Percentage are given, to find the Rate. Since the Percentage = the Base × the Rate, the Rate = the Percentage ÷ the Base. $15 \div 300 = .05$ (5%), the required per cent.

Or, 15 is $\frac{15}{300}$ or $\frac{1}{20}$ of 300. $\frac{1}{20} = \frac{5}{100}$, or 5%

Ex. What is 4% of £247 13*s.* 6*d.*?

OPERATION.

£	*s.*	*d.*
247	13	6
		.04
£ 9	.88	.52 .24
	20	
s. 18	.12	
	12	
d. 1	.68	

ANALYSIS.—Multiply the number of each denomination by .04, as in the margin, and then reduce the decimal parts to integers of lower denominations (**201**).

Or, reduce shillings and pence to the decimal of a pound (see note, Ex. 7, Art. 204), take the required per cent., and reduce the decimal result to lower denominations. Thus,

£247 13*s.* 6*d.* = £247.675
£247.675 × .04 = £9.907 = £9 18*s.* 1.68*d.*

270. These principles may be expressed by the following formulæ:

$$P = B \times R;\ B = P \div R;\ R = P \div B.$$

271. RULES.—*1. To find the percentage, multiply the base by the rate expressed decimally.*

the base, divide the percentage by the rate
mally.
the rate, divide the percentage by the base.

ding the rate, to produce a quotient of hundredths, make
of the dividend exceed those of the divisor by 2.

the rate is an aliquot part of 100, it is generally
t to use the equivalent fraction. Thus,

$\frac{1}{2}$.	$16\frac{2}{3}\% = .16\frac{2}{3} = \frac{1}{6}$.	$6\frac{1}{4}\% = .06\frac{1}{4} = \frac{1}{16}$.
$\frac{1}{3}$.	$12\frac{1}{2}\% = .12\frac{1}{2} = \frac{1}{8}$.	$5\% = .05 = \frac{1}{20}$.
$\frac{1}{4}$.	$10\% = .10 = \frac{1}{10}$.	$3\frac{1}{3}\% = .03\frac{1}{3} = \frac{1}{30}$.
$\frac{1}{5}$.	$8\frac{1}{3}\% = .08\frac{1}{3} = \frac{1}{12}$.	$2\frac{1}{2}\% = .02\frac{1}{2} = \frac{1}{40}$.

EXAMPLES.

is

?

.56 ?

80 ?

424 ?

00 ?

Find

6. 16% of \$375.

7. 8% of \$24.25.

8. $2\frac{1}{2}\%$ ($\frac{1}{4}$ of 10%) of 876.

9. $7\frac{1}{2}\%$ (10%—$\frac{1}{4}$ of 10%) of \$1678.

10. $\frac{3}{4}\%$ (1% — $\frac{1}{4}\%$) of \$21275.

s the difference between $2\frac{1}{2}\%$ of \$16000 and 5% of

hant bought goods amounting to \$375.60, and sold
ain 30% of the cost; how much did he gain?
r collected \$2875, and charged 5% for his services;
he retain for his services, and how much did he

14. What is the duty on twelve watches valued at \$75 each, at 25% of the value?

15. Jan. 10, a merchant buys a bill of goods amounting to \$876.40 on the following terms: 4 months, or less 5% if paid in 10 days. How much would settle the bill Jan. 18?

16. The product of two factors is 75; if one of the factors is .03, what is the other factor?

17. The percentage is 60, and the rate $2\frac{1}{2}\%$; what is the base?

18. \$18.08 are 5% of what?

19. \$324 are 3% of what?

20. \$37.56 are $2\frac{1}{2}\%$ of what?

21. \$17.28 are 24% of what?

22. 165 *ft.* are $33\frac{1}{3}\%$ of what?

23. £240 are $3\frac{1}{3}\%$ of what?

24. \$12.25 are $6\frac{1}{4}\%$ of what?

25. 96 francs are $\frac{4}{5}\%$ of what?

26. An agent sells a house and lot for $16450, and receives 5% for his services; what does he pay to the owner of the property?

27. Mr. A invests 42% of his capital in real estate, and has $53070 left; what is his capital?

28. If a man fails to pay his tax until he is charged 8% additional, how much will he lose if his tax is $36.75?

29. If the rate is 20% and the percentage 440, what is the base?

30. A has 35% of his property invested in stocks, 10% in horses and cattle, 18% in grain, and the remainder, which is $24235, in real estate. What is the total value of his property?

31. A merchant, failing in business, pays 43% of his indebtedness; he owes A $3750, and B $6280; how much does he pay each?

32. The product of two numbers is 375; if one of the numbers is 30000, what is the other number? Express answer in hundredths.

33. The assets of a bankrupt are $27387, and his liabilities $82161; what % of his indebtedness can he pay?

What per cent. of

34. 375 is 75?
35. $1728 is $144?
36. $3456 is $72?
37. 5280 *ft.* is 165 *ft.*?
38. $1000 is $12.50?
39. $3720 is $232.50?
40. $2416 is $60.40?
41. $1484 is $21.20?

42. A merchant paid for goods $345 and sold them for $258.75; the loss is what % of the cost?

43. If a paymaster receives $150000 from the treasury, and fails to account for $225 thereof, what is the percentage of loss to the government?

44. Total imports and exports carried in foreign vessels during the fiscal year 1858, were valued at $160,666,267; in American vessels for the same time, $447,191,304. What per cent. were carried in American vessels? In foreign vessels?

45. $640 being increased by a certain % of itself equals $720; required the rate %.

46. A commission merchant sold 450 barrels of flour at $5.30 per barrel; how much should he send to the miller, if he charges 2½ per cent. for making the sale?

47. A horse was sold for $658, which was 16⅔% more than it cost; what was the cost?

NOTE.—The cost of the horse was $\frac{100}{100}$, or 100% of itself; since the gain was 16⅔% of the cost, the selling price (the cost plus the gain) was 116⅔% of the cost. $658 is 116⅔% of what number?

What number increased by	What number decreased by
48. 25% of itself is 500?	*51.* 5% of itself is $307.80?
49. 8% of itself is $1004.40?	*52.* 40% of itself is 3726?
50. 125% of itself is 999?	*53.* 25% of itself is $342.60?

54. When the premium on gold was 17⅜%, what amount of gold was it necessary to sell to pay a note of $3000 in currency?

55. What is 116% of 1200?

56. 144 is 120% of what number?

57. 375 is what % of 300?

58. Find 95% of $1260.

59. Of what number is 275, 100%?

60. $187.50 are 2½% of what?

61. Total imports and exports carried in foreign vessels for the fiscal year, 1879, were valued at $911,269,232; in American vessels for the same time, $272,015,697. What per cent. were carried in American vessels?

62. The total tonnage entered at ports of the United States during the year ended June 30, 1879, was 13,768,137 tons. What per cent. was entered at the port of New York? (See Ex. 64.)

63. The tonnage entered at the four ports of New York, Baltimore, Philadelphia, and Boston, for the year ended June 30, 1879, was 10,489,660 tons. This amount constituted what per cent. of the total tonnage entered at ports of the United States? (See Ex. 62.)

64. The total tonnage entered at New York during the year ended June 30, 1878, was 5,545,026 tons; during the year ended June 30, 1879, 6,661,825 tons. What was the increase per cent.?

65. The earnings of the Chesapeake and Ohio R.R. Co. for the month of July, 1878, were $14,026,189; for the month of July, 1879, $17,338,273. What was the per cent. of increase?

66. Find 5% of £375.	*69.* Find 10% of £37 8*s.* 9*d.*
67. Find 2½% of £64 16*s.*	*70.* 16*s.* is 2½% of what?
68. Find 4% of £75 12*s.* 6*d.*	*71.* £1 8*s.* 4*d.* is 4% of what?

DISCOUNTS.

274. It is customary in many branches of business for manufacturers and dealers to have fixed price-lists of certain kinds of merchandise; and when the value changes, instead of changing a long price-list, the rate of discount is changed. The fixed price is called the *List-Price*, and the discount allowed the *Trade Discount*.

Books are usually sold by publishers and jobbers at certain discounts from the retail prices.

275. Many kinds of merchandise are sold at "time" prices, subject to certain rates of discount if paid at an earlier period.

1. Thus, the following or similar announcements are usually found upon the bill-heads of wholesale dealers: "Terms, 4 months, or 30 days, less 5%"; or, "Terms 60 days, or 1% discount in 30 days, or 2% discount in 10 days."

2. In the same business house, certain goods are sold on long credit, and others on short credit.

3. When no rate of discount has been offered, merchants are generally willing, when bills are paid before maturity, to deduct the interest on the amount of the bill for the remainder of the time at the legal rate per annum.

Ex. The list-price of a scale is \$80; what is the net price if a discount of 25% and 10% is allowed?

OPERATION.

\$80 List-price.
20 25%, or $\frac{1}{4}$.
60
6 10%, or $\frac{1}{10}$.
54 Net-price.

ANALYSIS.—The first rate of discount is reckoned upon, and deducted from the list-price, and the others are deducted from the successive remainders.

The result is not affected by the order in which the discounts are taken. A discount of 25% and 10% is the same as a discount of 10% and 25%.

EXAMPLES.

276. *1.* The gross amount of a bill of shoes is \$82.68. What is the net amount, the rate of discount being 5%?

2. A stove is sold for \$45 less 30%; required the net price?

NOTE.—If the discount is not required, multiply by .70 (100% − 30%); the product will be the net price.

3. What is the value of 466 *lb.* O. W. casing @ 45 *cts.* per pound, less $1\frac{1}{2}$ per cent.?

4. The gross amount of a bill of mdse. is $106.36; what is the net amount, the rates of discount being 20% and 10%?

5. The gross amount of a bill of notions is $49.75; what is the net amount, the rates of discount being 10% and 10%?

6. What is the value of 12 pair shoes @ $1.60 per pair, less 5%?

7. What direct discount is equivalent to a discount of 15% and 10%? 45% and 10%? 20% and 12½%? 60% and 10%? 75% and 12½%?

8. What is the net value of one case prints containing 2273 *yd.*, @ 4³ *cts.*, less 5%, cooperage 25 *cts.*?

9. A bill of merchandise amounting to $442.38 was bought Aug. 18, 1879, on the following time: "4 months or 5% off 30 days." How much would settle the bill Sept. 16, 1879?

10. What is the net value of a bill of iron amounting to $1103.75, at a discount of 45, 10, and 2 per cent.?

11. What is the net value of 1 case prints containing 3039² *yd.* @ 5 *cts.* per *yd.*, less a discount of 3%; cooperage $.25?

12. The net amount of a bill of files was $36.75; what was the gross amount, the rate of discount being 10%?

13. Mr. A. is offered dress goods at 26² *cts.* per *yd.*, "4 months, or less 6% cash"; how many yards can he purchase for $49.82?

14. The net amount of a bill of hardware is $175.26; what is the gross amount, the rate of discount being 45% and 10%?

15. What is the difference on a bill of $875 between a discount of 40% and a discount of 30% and 10%?

16. A bill of tinware is sold at the following discounts: $74.20 at 20% and 10%; $43.75 at 40% and 5%; $69 at 33⅓% and 10%; and $49.17 net. What is the total net amount of the bill?

17. A bill of dry goods amounting to $914.37 is sold, Aug. 19, on the following terms: "60 days, or less 1% if paid in 30 days, or less 2% if paid in 10 days." How much would settle the bill Sept. 18? How much Aug. 27?

18. Of a bill of hardware, $61.51 are sold at a discount of 60 and 5%; $18.75 at a discount of 10%; $16.86 at a discount of 12½%; $44.25 at a discount of 40 and 5%; $29.60 at a discount of 40, 12½, and 10%; $28.04 at a discount of 55%; $16 at a discount of 65, 10, and 10%; $18.70 at a discount of 50%; $19.75 at a discount of 20%; $18.50 at a discount of 15%; $307.55 at a discount of 75 and 12½%; $36.61 at a discount of 60 and 10%; and $218.25 net. What is the total net amount of the bill?

BILLS.

277. A **Bill** is a detailed statement of merchandise sold, or of services rendered. Bills of merchandise state the place and date of the sale, the names of the buyer and seller, the terms of the sale, the quantity, price, and distinguishing marks and numbers of the merchandise, and other details.

The terms *Bill* and *Invoice* are used by many interchangeably. The term Invoice is applied more particularly to statements rendered by consignees to commission merchants, showing marks, numbers, values, and accrued charges of goods shipped; to bills rendered to jobbers; and to bills received from foreign countries.

EXAMPLES.

278. Copy the following bills, make the necessary extensions, and find the total amounts:

(1. Canned Goods.)

Folio 316. WILMINGTON, DEL., *Nov. 16, 1876.*

Messrs. WM. DOLTON & Co.,

Bought of JAMES MORROW & SON.

Cases.	*Doz.*					
2	4	3 lb. Peaches	$2\frac{25}{}$	9	00	
1	2	2 " Saco Corn	$1\frac{85}{}$	*	**	
1	2	$2\frac{1}{2}$ " Salmon	$3\frac{85}{}$	*	**	
2	4	3 " Tomatoes, B. & L.	$1\frac{80}{}$	*	**	
1	2	$2\frac{1}{2}$ " Col. Pears	$4\frac{00}{}$	*	**	
1	2	$2\frac{1}{2}$ " Apricots	$4\frac{00}{}$	*	**	
		Ctge.			50	$**	**
		P. R. R.				

(2. Flour.)

BUFFALO, N. Y., *Dec. 6, 1880.*

Messrs. DANIEL CROUSE & SONS,

Bought of SCHOELLKOPF & MATTHEWS.

Interest charged on all accounts after 30 days. We allow no Expressage or Exchange.

	20	Bbls. Flour "Sunlight" Sacks	$7.05	***	**	
	25	" " " Bbls.	7.25	***	**	
	25	" " "Victor" Sacks	6.05	***	**	
	25	" " " Bbls.	6 25	***	**	
	15	" " "Dakota" Sacks	5.30	**	**	
	5	" " "Superlative" Sacks	8.55	**	**	
20 bags		2177 lb. S. Meal	1.20	**	**	
70 "		$264\frac{9}{32}$ bu. Oats	.56	***		***	**
90 total.						

(3. Storage, etc.)

BROOKLYN, N. Y., *Jan. 30, 1879.*

BARK ENTERPRISE,

To J. P. & G. C. ROBINSON, *Dr.*

Terms Cash.

Storage	16319^{23}	bu.	@	$\frac{3}{4}¢$	122	39		
Elevating	16319^{23}	"	@	$\frac{1}{2}¢$	**	**		
Delivering	16319^{23}	"	@	$\frac{1}{2}¢$	**	**		
Weighing	16319^{23}	"	@	$\frac{1}{2}¢$	**	**		
Carting	16319^{23}	"	@	1¢	***	**		
Loading ship	16301^{34}	"	per M.	$7.00	***	**		
Separating damage	16301^{34}	"	@	$\frac{1}{2}¢$	**	**		
Blowing on delivery	16301^{34}	"	@	$\frac{1}{4}¢$	**	**		
Weighing on "	16301^{34}	"	@	$\frac{1}{2}¢$	**	**	***	**

(4. Provisions.)

CLEVELAND, O., *Oct. 9, 1876.*

Messrs. L. C. MAGAW & SON,

Bought of J. P. ROBISON & Co.

Terms Cash.—*No goods sold on 30 days.*

10	Bbls. S. M. Pork				$17.00	***		
5	" Mess Beef				$10.75	**	**	
5	" Hams	90[a]	1376[b]	—98[c]	****[d]	14¢	***	**
3	" Shoulders	58	744	—57	***	9¢	**	**
1	" Dr. Beef	33	241	—22	***	14¢	**	**
1	Tc. Lard		406	—60	***	11¢	**	**	***	**

[a] Number of pieces. [b] Gross weight. [c] Tare, or weight of barrel or tierce. [d] Net weight.

(5. Fish.)

GLOUCESTER, MASS., *Sept. 28, 1876.*

Messrs. DANIEL WEIDMAN & Co.,

Bought of CLARK & SOMES.

Subject to sight draft without notice after thirty days.

2	Qtl. New Geo. Cod	5.75	**	**	
1	Bbl. Ex. $1 Mackerel	20.00	**	**	
10	Kits 15 lbs. Ex. $1 Mackerel	1.80	**	**	
10	" 20 lbs. Bay $1 "	1.80	**	**	
2	Bbls. $2 Shore " lg.	12.00	**	**	
10	Kits 20 lbs. $2 Shore " "	1.50	**	**	
5	Halfs New Labrador Herring	3.82	**	**	
3	" Round Shore "	2.95	*	**	
	Box .88, ctg. in Boston .90		*	**	$***	**

(6. Groceries.)

Order Book, 410-22.
Day Book, 115-197.

NEW YORK, *Feb. 1, 1880.*

Messrs. EDWARDS & Co.,

Bought of H. K. & F. B. THURBER & Co.

Terms Cash 30 days.
Shipped per National Line.

When you desire to order goods, same as had before, give date of purchase, and the Order and Day Book pages.

M P C $4385	1	Cask Old Prunes 1544 — 134 = **** lbs. -	4¾	** **
	3	Boxes Old Muscatel Raisins - - - - - -	1⁶²⁵	* **
	3	" New " " - - - - - -	2¹⁰	* **
	4	" " Layer " - - - - - -	1²⁵	* **
	1	" Cream Tartar, ¼ foil - - 20 lbs. -	.39	* **
	2	" Yeast-Cakes, 3 doz. ea., - 6 doz. -	.65	* **
	25	lbs. Whole Pepper - - - - - - - - -	.16	*
	10	" Nutmegs $1 - - - - - - - - -	1⁰⁰	**
		Bag -		15
	1	Box O. K. Mustard, ¼'s - - - 12 lbs. -	.25	*
	1	" " " ½'s - - - 12 " -	.25	*
		Cartage on all - -	1	
			***	**

(7. Groceries.)

NEW YORK, *Aug. 13, 1876.*

Messrs. HORTON, CRARY & Co.,

Bought of AUSTIN, NICHOLS & Co.

W. B. A $99	1	Bag ·20 Rio Coffee - - - - - - -	132	23	30 56
	1	" ·20 " - - - - - - -	131	21½	** **
	1	Bbl. ·25 Roa. Java Coffee 121 / 21 -	100	25½	** **
	2	" ·50 " Rio " 112—22 221 / 109—20 42 -	***	24	** **
H. R. P.	1	Case Conc. Lye - - - - - - - - - -			5 50
Union.	2	Boxes Yeast Cakes, ea. 3 - - - -	*	65	* **
	25	lbs. Spice, Bag 20¢ - - - - - -		15½	* **
	5	Mats Cassia - - - - - - - - -	21½	26	* **
A. N. & Co.	1	Keg Gr. Mustard - - - - - - -	50	35	** **
	10	lbs. White Glue - - - - - - - - -		40	*
A. N. & Co.	5	Bbls. X. C. Sugar - - 257—20 **** / 269—20 ** / 256—21 — / 253—18 / 253—20 -	****	11¾	*** **
$134	1	" W. D. Syrup - - - - 47 ½	***	60	** **
$114	1	" C. D. " - - - - 45 ¼	***	50	** **
		Ctg. - - -			1 50
		Syrup, 60 days - - **.**			$*** **
		Balance, 30 " - - ***.**			
		***.**			

(8. Dry Goods.)

NEW YORK, *March 20, 1879.*

Messrs. FIELD, LEITER & Co.,

Bought of H. B. CLAFLIN & Co.

Terms Cash in 30 days less 5%, or 4 months' note delivered within 30 days, and payable at Bank in New York exchange.

2875	1	Bale Boott M. Brown	800	06^{3}	54	
8039	1	" Continental C. do.	800	07^{1}	**	
3369	1	" Pequot A. 36 in.	967	07^{2}	**	**
1290	1	" Great Falls E.	1111	07^{1}	**	**
1590	1	" Atlantic H. 1038 .07^{3}	$ **	**		
		Less 4%	*	**	**	**
6888	1	" Boott F. F.	800	07^{1}	**	
2179	1	" Pepperell 600 Drill	622	07^{3}	**	**
2507	1	Case Blackstone A. A.	1649	07^{1}	***	**
6515	1	" Dwight Anchor	1139	09	***	**
2985	1	" Great Falls Q.	1492	08	***	**
1650	1	" Pearl River Ticking	708	15^{2}	***	**
		Cooperage				75
					***	**

(9. Dry Goods.)

NEW YORK, *March 23, 1878.*

Messrs. DAVIDGE, LANDFIELD & Co.,

Bought of TEFFT, GRISWOLD & Co.

2	Naumkeag Bl. Jean 48/47	95	09	8	55
4	Roll Cambric 46 46/40¹ 46	****	05^{2}	*	**
3	Pepperell Drill 47³/30³ 44³	****	08	*	**
1	Lowell $^{10}/_{4}$ Brown	38	14^{2}	*	**
3	Continental C. 40/40/40	***	07^{3}	*	
5	New Market N. 45³ 45/45¹ 58¹/46³	****	06^{1}	**	**
2	Champion Cheviot 48¹/50¹	***	09	*	**
2	Otis B. B. Dk Stripe 57³/57¹	***	10	**	**
1	Hamilton 30 in. Tick	48^{3}	11^{2}	*	**
2	Thorndike C. 58³/61	****	08^{2}	**	**
2	Wamsutta C. Blea. 58¹/68¹	****	12	**	**
8	Andros L. 52 52 49 51¹/51 51³ 51 52²	***	07^{3}	**	**
1	Pepperell $^{10}/_{4}$	36^{3}	22	*	**
				***	**
	Cooperage			1	25
				***	**

(10. Dry Goods.)

Book 174, Page 148. NEW YORK, *March 30, 1878.*

Mr. JAMES MORGAN, *Milwaukee, Wis.*

Bought of H. B. CLAFLIN & Co.

Terms: Net 60 Days, or 1% discount in 30 days, or 2% discount in 10 days, N. Y. Funds. No Exchange allowed.

‡4641	53	Pc's Gordon Prints (Job)				
		21^{2} 48^{2} 38 40^{1} 48^{2} 48^{3} 37^{2} 48 48				
		44 49^{2} 44^{3} 48^{2} 49^{3} 49^{3} 49^{2} 42 56				
		48^{2} 49^{1} 28^{2} 49^{1} 49 48^{3} 49^{1} 28 48^{3}				
		37 33^{2} 49^{2} 52 33^{3} 40 48 49^{1} 49^{1}				
		24 48^{2} 48^{2} 52 48^{3} 49 47^{2} 48^{1} 48^{2}				
		49^{1} 49^{2} 48^{3} 48^{2} 48^{2} 43^{2} 49^{1} 49^{2} -	*****			
‡2601	54	Pc's Do.				
		48^{3} 48 49 42 22^{1} 49^{1} 49 48^{2} 53^{2}				
		48^{2} 47^{3} 48^{3} 48^{2} 49 44 49 49^{2} 48^{2}				
		49^{2} 49 49 48^{2} 47^{3} 47 48^{2} 49^{1} 56				
		50^{2} 49^{1} 41^{1} 48^{1} 50 27^{1} 49 48^{2} 48^{3}				
		21^{3} 29^{1} 51^{3} 46^{3} 48^{2} 48^{2} 28^{2} 48^{2} 49^{1}				
		49^{2} 45^{2} 47 48^{2} 40^{2} 50^{1} 39^{2} 48^{2} 46^{1}	*****			
‡4765	61	Pc's Do.				
		30^{2} 49^{2} 42 49^{2} 32 48 46 48^{2} 46^{2}				
		42^{3} 47^{2} 22^{1} 33 46 48 49^{2} 48^{2} 48				
		42 42 48 28 48^{1} 49^{2} 48^{2} 49 49				
		49^{2} 48^{2} 28^{2} 49^{2} 43 49^{1} 48^{2} 49^{2} 48				
		38^{2} 29 25 26^{3} 49^{1} 49^{3} 49^{1} 49 48^{2}				
		34^{3} 48^{3} 45 49 49^{1} 49^{2} 48^{1} 36 48				
		29^{2} 49^{3} 48^{2} 31^{1} 48^{2} 49 48^{1} - -	*****			
			*****	$.04^{2}$	***	**
		Cooperage -			1	00
					***	**

(11. Dry Goods.)

NEW YORK, *March 20, 1878.*

Messrs. JORDAN, MARSH & Co.

Bought of A. T. STEWART & Co.

Job.	8	Cases Gordon Fancy					
J. U.		‡4561 2810					
S. B. R.		4157 2902^{1}					
H. Z.		3473 2787^{2}					
S. J. L.		4224 2880^{2}					
G. Q.		2777 2821^{1}					
J. B.		3504 2842^{2}					
J. Z.		3970 2883^{1}					
J. H.		4198 2863^{1} - - ****** .05			****	**	
		Less 5% -			**	**	****	**

(12. Hosiery.)

Claims for Damages or Errors must be made on receipt of Goods.

NEW YORK, *June 28, 1880.*

Messrs. JOHN FORD, SONS & Co.,

Bought of JAMES TALCOTT.

Net 30 Days.
Note to your own order
Payable at a Bank in New York City.

1789	35	Doz. 3458 Mixed ½ Hose - -	.80	28			
	25	" 2032 Fancy " " - - -	.80	**			
	12	" 853 Col'd " - - -	1.00	**			
	12	" 1691 Fancy " - - -	1.00	**			
	18	" 1759 " " - - -	.75	**	**		
	20	" 1713 " " - - -	1.00	**			
	16	" 1716 " " - - -	1.10	**	**		
	6	" 3438 Fch mx ½ " - - -	.90	*	**		
	22	" Job Misses " - - -	.75	**	**	$***	**
		Shipped per P.R.R.& C.B. & Q.R.R.					

(13. Books.)

CHICAGO, ILL., *May 7, 1878.*

Mr. JOHN BERWOLD,

Bought of HADLEY BROS.

Terms Cash.

12	Randall's Arithmetics, Part 1 -	.60	7	20		
18	" " " 2 -	.50	*			
24	Smith's Primers (paper) - - -	.06	*	**		
36	" Spellers - - - - - -	.22	*	**		
18	" 2d Readers - - - - -	.45	*	**		
12	" 3d " - - - - - -	.70	*	**		
6	" 4th " - - - - - -	1.15	*	**		
6	" 5th " - - - - - -	1.35	*	**		
6	Doz. Brown's Copy Books - - -	1.80	**	**		
			**	**		
	Less 33⅓ % -		**	**	**	**
6	Jones' Geographies $1 - - - -	.35	2	10		
6	" " 2 - - - -	.63	*	**		
6	" " 3 - - - -	1.10	*	**		
6	" " 4 - - - -	2.00	**			
			**	**		
	Less 25 % -		*	**	**	**
3	Boxes Chalk Crayons - - - -	.18				**
3	Doz. Blank Copy Books - - -	.50			*	**
					$**	**

(14. Hardware.)

PHILADELPHIA, PA., *Aug. 13, 1880.*

Messrs. N. RUTTER, SON, & Co.,

Bought of BIDDLE HARDWARE CO.

Terms 60 days.

24	Sets W'd Wh'l Bed Casters $1 2 in. - .18	*	**		
	50% - -			*	**
1	Doz. Russell's S.B. Knives 14 in. $1540 -			11	
200	Carriage Bolts ¼ × 1 (2.40) 2¼ (2.55) 5¼ (3.15) 5½ (3.20)	**	**		
100	" " 5/16 × 5¼ (4.50) 5¾ (4.70) 6¼ (4.90) 7¼ (5.30)	**	**		
100	" " ⅜ × 5¼ (5.95) 5 (6.25) 6¼ (6.50) 6¾ (6.85)	**	**		
100	" " ⅜ × 7¼ (7.15) 7¾ (7.45) - - -	**	**		
100	" " ⅜ × 8½ (7.90) 8¾ (8.05) - -	**	**		
100	" " 7/16 × 2½ (6.50) 4½ (8.00) 7½ (10.40) - -	**	**		
100	" " 7/16 × 8 (10.80) 8½ (11.20) - - - -	**	**		
100	" " ½ × 2 (7.25) 2½ (7.75) 4 (9.25) - -	**	**		
100	" " ½ × 6 (11.25) 6½ (11.75) 8 (13.25) - -	**	**		
		***	**		
	75 & 12½% -			**	**
½	C. Machine Bolts ½ × 8 8.70	*	**		
½	" " " ¾ × 6 (15.10) 7 (16.60) - - -	**	**		
99	lbs. " " ¾ × 11 .10¾	**	**		
		**	**		
	60 & 10% -			**	**
	5 Cases Packing and Cartage -			1	
				**	**

(15. Watches and Jewelry.)

NEW YORK, *Mar. 7, 1877.*

Mr. CHARLES BABCOCK,

Bought of WHEELER, PARSONS & HAYES.

Terms: Net Cash 4 months, or less 5% 30 days, with Exchange on New York.

H 658	1	18 k. Ancre 17 L. full Engrd & Enld S. W.	90			
20422	1	14 k. Russell 17 L. flat C. B. "	46	50		
	1	18 k. Plain Ring 3¾ dwts. @ 1.05 - - -	*	**		
			***	**	Gold	
		Premium 5½% -	*	**		
	2	14 k. Guards with slides 37¾/222, 56/206 @ 1.15 -	***	**		
	1	Pr. Solid Roman Sl. Buttons 908 - - -	10	50	***	**

(16. Tinware.)

ROCHESTER, N. Y., *Oct. 16, 1880.*

Messrs. MCCARTHY & REDFIELD,

Bought of JOHN H. HILL.

Terms 60 days. If paid in 10 days 2 per cent. discount.

	2	Doz. #21 Pieced Dish Pans - -	8.25	**	**		
	½	" 9 in. Wash Boilers - - -	36.00	**			
	3	" Pieced Bread Pans 3×9×3 -	2.00	*			
	3	" " " " 5×9×2 -	2.00	*			
	1	" #5 Pieced Covered Pails -		2	50		
	3	" #13 " Cups - - - -	.90	*	**		
	1	" #15 " Dippers - - -		1	25		
	2	" #25 " " - - -	1.75	*	**		
	6	Nests #021 Flaring P'ls & Dippers	1.14	*	**		
				**	**		
		20 & 12½% -		**	**	**	**
	1	Doz. Champion Nutmeg Graters				1	75
1 Case	.15	1 " Nests #4 Fancy Cov'd Pails		6	00		15
1 "	.17	1 " #4 Burnished Tea Pots - -		6	75		17
				**	**		
		25 & 12½% -		*	**	*	**
	1	" #9 Pudding Pans - - - -		3	50		
	2	" #10 " " - - - -	4.25	*	**		
	½	" #200 Pressed Kettles - -	5.50	*	**		
	1	" #220 " " - -		7			
				**	**		
		37½% -		*	**	**	**
	6	2 qt. En'ld Bel. Sauce Pans - -	.63	*	**		
	6	3 " " " " "	.73	*	**		
				*	**		
		40% -		*	**	*	**
		.75 .90					
	6	Enameled Kettles Ea. 4—5 qt. -		*	**		
		1.10 1.30					
	12	" " 6—8 qt. -		**	**		
				**	**		
		60% -		**	**	**	**
1 Crate	7	8—W. H. Tea Kettles - - -	.95	*	**		
		45% -		*	**	*	**
		4 Boxes 2.06, Carting .38 -				*	**
						**	**
		N. Y. C. & H. R. R. 975 lbs. @ 12¢					

COMMISSION AND BROKERAGE.

279. **Commission** or **Brokerage** is an allowance made to an agent for transacting business for another; as, the sale or purchase of property, the collection or investment of money, etc.

An additional percentage is usually charged by commission merchants for guaranteeing the payment of sales made on credit.

280. The party who transacts the business is called a **Commission Merchant**, or **Broker**; and the one for whom he acts is called a **Principal**.

NOTES.—1. Commission Merchants usually have possession of the subject-matter of the negotiation, and make sales and purchases in their own name.

2. Brokers do not have possession of the merchandise bought or sold, and generally make contracts in the names of those who employ them and not in their own. They simply effect bargains and contracts.

The name *broker* is often erroneously applied to dealers in stocks, bonds, etc., who buy and sell on their own account only.

281. A **Consignment** is a quantity of merchandise sent by one party to another. The party who sends it is called the **Consignor**; and the party to whom it is sent, the **Consignee**.

282. The **Net Proceeds** of a consignment is the balance due the consignor after all charges or expenses have been deducted.

The whole amount realized from a sale is called the *gross proceeds*. The commission is usually a certain per cent. of this amount.

283. An **Account Sales** is a detailed statement rendered by the Commission Merchant to the Consignor, showing the sales of certain goods, the charges or expenses attending the same, and the difference or net proceeds.

The charges embrace freight, cartage, inspection, advertising, storage, insurance, commission and guarantee, etc.

284. An **Account Purchase** is a detailed statement rendered by the Commission Merchant to his Principal, showing the cost of certain goods, and the charges or expenses attending the purchase.

285. Commission or brokerage is usually computed at a certain per cent. of the amount realized or invested, or of the amount

involved in the transaction. In such cases the general principles of percentage are applied.

NOTES.—1. In buying and selling stocks, bonds, etc., the par value, and not the actual value, is taken as the base.

2. The commission for buying and selling some kinds of merchandise is usually computed at a certain price per unit of weight or measurement; as, grain per bushel, cotton per bale, etc.

EXAMPLES.

286. *1.* A commission merchant sold goods to the amount of $864; what was his commission at 2½ (¼ of 10) %?

2. A salesman sells goods at a commission of 2½%; what must be his sales, that he may have a yearly income of $5000?

3. What is the brokerage for selling 850 bales of cotton at the rate of $25 per 100 bales?

4. A lawyer collected a note of $2375; how much did he pay to the owner of the note, his commission being 5%?

5. My agent in Chicago purchases for me 600 barrels of flour at $3.75 per barrel; how much do I owe him, his commission for purchasing being 2%?

6. An officer collected $17850, and deposited $17493 in the Treasury, retaining the remainder as his commission. What was the rate per cent. of the commission?

7. Sent to a commission merchant in Toledo $2080.80 to invest in flour, his commission being 2% on the amount expended; how many barrels of flour would be purchased at $4.25 per barrel?

8. A commission merchant sells merchandise amounting to $3325; how much is paid to the consignor of the merchandise, the charges being, for transportation $117.50, for advertising $10, for storage $15, for commission 2½%?

9. My agent in Chicago buys for me 1187.76 centals wheat at $2.123 per cental. What is his commission at ½ per cent.?

10. A commission merchant purchased for me $9\frac{28}{}$ bushels of clover seed at $8.55 per bushel. How much should I send to him in settlement, if his commission for purchasing is 1 per cent.?

11. A broker buys 8375 pounds of leather at 26 cents per pound. What is his brokerage at ¼%, and what is the net amount received by the seller, the brokerage being paid by him?

12. A freight broker procures transportation for 375 tons of merchandise at $3.50 per ton; what is his brokerage at 5%?

13. A collector deposits $28117, retaining 3% on the whole amount collected. What amount did he collect and what was his commission?

14. A lawyer, collecting a note at a commission of 5% thereon, received $6.25; what was the face of the note?

15. An agent sold 6 mowing-machines at $120 each, and 12 at $140 each. He paid for transportation $72, and, after deducting his commission, remitted $2208 to the manufacturer. What was the % of his commission?

16. A merchant instructs his agent in Cincinnati to buy pork to the amount of $5000. The charges on the pork being $16, and the agent's commission $1\frac{1}{2}\%$, how much must be remitted to settle the bill?

17. What are the net proceeds of the sale of 12372 pounds of leather at 22 cents per pound, the charges being $31, and a commission of $2\frac{1}{2}\%$ being paid for selling and $2\frac{1}{2}\%$ for guaranteeing payment?

18. A real estate agent, who charged $2\frac{1}{2}\%$ for making the sale, paid to the owner of a house and lot $42412.50; what was the value of the property?

19. A commission merchant sells 240 *bbl.* of potatoes at $3.75 per *bbl.*, and 260 *bbl.* at $3.60 per *bbl.* How much is due the consignor, the commission being $12\frac{1}{2}$ cents per barrel?

20. John Smith is a disbursing agent of the United States. Jan. 1, 1880, there is in his hands $11870.63. Feb. 1, he pays out $3220.34, on which he is entitled to a commission of $1\frac{1}{4}\%$. Mar. 1, he receives $3750.87. May 1, he pays out $3795.01, on which he is entitled to a commission of $2\frac{1}{2}\%$. Make a statement of his account, showing balance due the United States.

21. What are the proceeds in currency of $2611.06 gold, at $1.06\frac{5}{8}$, commission for selling $\frac{1}{16}\%$?

22. A, having a claim against the government of $10970, agreed to pay an agent 8 per cent. of the amount collected. But the amount collected was 22 per cent. less than the amount of the claim. How much was received by A?

23. B sends $2240.70 to his agent in Cleveland requesting him to invest in provisions after deducting his commission for purchasing of 3%; what was the sum invested?

24. A broker received $62.50 for selling some bonds, charging $\frac{1}{8}\%$ brokerage. What was the par value of the bonds?

25. A commission merchant sold 300 bales of cotton, averaging 462 *lb.* to the bale, at 15.7¢, his commission being 25¢ per bale, and the charges $161. He purchased for the consignor dry goods amounting to $2576.37, charging a commission of 1½%. How much was still due the consignor?

26. A of Chicago, sends to B of New Orleans, 8000 *bu.* of wheat and 500 *bbls.* of flour with instructions to sell it and invest the proceeds in sugar. B pays freight and cartage $3420; sells the wheat at $1.60 per bushel and the flour at $5.25 per barrel; charges 2½% commission on the flour and 1¢ per bushel on the wheat: how many pounds of sugar are purchased at 8½ cents per pound, the commission for purchasing being 3%?

Copy the following accounts, and make the necessary extension, etc.

(27. Account Sales.)

NEW YORK, *Oct. 19, 1880.*

Sold for account of A. W. RANDOLPH & Co.,

By DAVID DOWS & Co.

1880.							
Sept.	12	100 Bbls. "Sunshine" - - - -	5.75	***			
"	18	125 " "Pride of the West" -	6.25	***	***		
"	30	150 " "Sunshine" - - - -	6.	***			
Oct.	14	75 " "Pride of the West" -	6.50	***	**		
"	18	50 " " " -	6.60	***			**
		Charges.					
Sept.	10	Transportation 500 Bbls. @ 27¢ - - -		***			
"	10	Cartage 400 " @ 5¢ - - -		**			
Oct.	19	Storage 400 " @ 3¢ - - -		**			
"	19	Insurance $\frac{1}{16}$% - - - - - - - -		*	**		
"	19	Commission and Guarantee 5% - - -		***	**	***	**
		Net proceeds - - - - - -				****	**

(28. Account Purchase.)

TOLEDO, O., *Mar. 6, 1877.*

Purchased by A. L. BACKUS & SONS.

For account and risk of L. A. & W. B. SHAW.

9	Bags "Montauk" - - - - - -	.21	*	**		
2^{27}	Bu. Mammoth Clover Seed - -	9^{00}	**	**		
9^{28}	" Clover Seed - - - - - -	8^{66}	**	**		
9^{31}	" Timothy Seed - - - - - -	1^{75}	**	**	***	**
	Charges.					
	Cartage - - - - - - - - -			25		
	Commission 1% - - - - - - -		*	**	*	**
	Charge your % - - - - - -				***	**

PROFIT AND LOSS.

287. Profit and Loss treats of the gains (profits) and losses which arise in business transactions.

The profit or loss is always estimated on the cost price, or the amount invested. Discounts are reckoned on the market or asking price. (See Art. **274.**)

288. The difference between the cost of goods and the price at which they are sold is a **profit** or a **loss**,—profit if the selling price is the greater, loss if the cost is the greater.

EXAMPLES.

289. *1.* A man purchased a horse for \$250, and sold it at a gain of 16%. What was the gain? (Gain = .16 × cost.)

2. A merchant sold goods that cost \$325 at an advance of 12%; what was the selling price? (Gain = .12 × cost, and selling price = cost + gain; or, selling price = 1.12 × cost.)

3. Bought a farm for \$3600, and sold it at an advance of 25%; what was the gain?

NOTE.—If, as in the above example, the rate per cent. is an aliquot part of 100, it is more convenient to use the equivalent fraction (**272**). Thus, 25% = .25 = ¼; gain = ¼ of cost.

4. Cloth is bought at \$6 per yard, and sold at a loss of 20%. What is the selling price? (Selling price = ⅘ of cost.)

5. Bought a house for \$3475; at what price must it be sold to gain 36%?

6. Purchased flour at \$6.25 per barrel; at what price must it be sold to gain 20%?

7. If I buy hats at \$27 per dozen, at what price must they be sold apiece to gain 33⅓%?

8. A factory which cost \$8775 was sold at a gain of 16%. What was received for it?

9. If silk costs \$1.68 per yard, and is sold at an advance of 12½%, what is the profit per yard?

10. A merchant purchased goods to the amount of \$8735, and sold them at a loss of 12%; what was his loss?

11. Bought 125 barrels of flour for \$600. If sold at an advance of 15%, what was the profit per barrel?

12. A lot of dry goods was sold at an advance of 18%. If the gain was $436.50, what was the cost? (Gain = .18 × cost; hence, gain ÷ .18 = cost.)

13. A farm was bought for $7200, and sold at a gain of $900; what was the gain per cent.? (Gain = gain % × cost; hence, gain % = gain ÷ cost.)

14. A man paid for merchandise $875, and sold it for $1015; what per cent. did he gain?

15. A man paid for merchandise $1015, and sold it for $875; what per cent. did he lose?

16. Find the rate % of profit on goods bought for $324 and sold for $364.50.

17. A painting was sold for $2343, at a gain of 32%; what was the cost? [Selling price = 1.32 (100% + 32%) × cost; hence, cost = selling price ÷ 1.32.]

18. Find the cost of goods sold at an advance of 12½%, being a profit of $76.

19. How much was paid for a farm sold for $9878, at 12% below cost?

20. What is the profit on iron sold for $4520, at an advance of 13% on cost?

21. What is the selling price of tea which cost 32 cents per pound and is sold at a profit of 37½%?

22. Sold drugs for $168, at an advance of 75%; what was the profit?

23. A merchant sold for $2576 a lot of dry goods for which he paid $3360. What was the per cent. of loss?

24. A mixture is made of 1 gallon of wine at 50 cents a gallon, 3 at 90 cents, 4 at $1.20, and 12 at 40 cents. What per cent. would be gained by selling the mixture at $1.60 a gallon?

25. If, by selling tea at 47½ cents per pound, I lose 5%, at what price must I sell it to gain 15%?

26. If, by selling goods for $126, I lose 16%, what per cent. would I have lost or gained if I had sold them for $168?

27. A merchant's price is 25% above cost price. If he allows a customer a discount of 12% on his bill, what per cent. profit does he make?

28. If cloth, when sold at a loss of 25%, brings $5 per yard, what would be the gain or loss per cent. if sold at $6.40 per yard?

29. Goods that cost $168 are sold at an advance of 25%; what is the selling price?

30. What must be the asking price of goods costing $32, that I may deduct 20% from it, and still gain 25% on the cost?

31. Sold a horse at a gain of 33⅓%, and with the proceeds purchased another horse, which I sold for $120 at a loss of 20%. What was the gain or loss?

32. What must ribbon be sold per yard so as to gain 20%, if 22½ yards cost $6.75?

33. Books are purchased at a discount of 30% from the list price (**274**). What is the gain per cent. by selling at the list price?

34. What per cent. is gained by selling pans at 21 cents apiece, that cost $2.56 per dozen less 20 and 12½%?

35. Plows are bought at a discount of 50% from the list price. What per cent is gained by selling at the list price?

36. A merchant purchases goods at a discount of 25% from the list price. What per cent. is gained by selling at the list price. What per cent. if goods are purchased at a discount of 33⅓%? 35%? 25% and 5%? 20% and 12½%? 15% and 10%?

37. A merchant's retail price for boots is $4.75 per pair, by which he makes a profit of 33⅓%. He sells to a wholesale customer at a discount of 20% from the retail price. What per cent. does he gain or lose, and what does he receive per pair?

38. 40 head of cattle weighing 52770 pounds are purchased in Chicago at $4 80 per *cwt.*, and are sold in New York at 10½ cents per pound, to dress 56 pounds. What is the gain per cent., making no allowance for transportation? What was the total cost? The total selling price?

NOTE.—The quantity bought or sold does not affect the gain or loss per cent.

39. A speculator sold two building lots for $4800 each. On one he gained 20%, and on the other he lost 20%. Did he gain or lose, and how much?

40. If a merchant buys goods at a certain price 10 and 5 off, and sells them at the same price, 5 off, what per cent. profit does he make?

41. What must be the asking price for books that cost $1.60, in order to abate 20%, and still make a profit of 25%?

INTEREST.

DEFINITIONS.

290. Interest is a sum charged for the use of money, or its equivalent.

291. The **Principal** is the sum for the use of which interest is charged.

292. The **Rate** is the per cent., or number of hundredths, of the principal, charged for its use for a certain time, usually for one year (per annum). When no time is mentioned with the rate in the contract, a year is understood.

293. The **Amount** is the sum of the principal and interest.

If $1000 is loaned for one year at 6% per annum, $60 would be the interest, $1000 the principal, and $1060 the amount.

294. Simple Interest is interest on the principal only for the full time.

295. Compound Interest is interest not only on the principal, but on the interest also after it becomes due.

If $1000 is loaned Jan. 1, 1881, for 2 years, the amount due Jan. 1, 1883, at 6% simple interest, would be $1000 (Principal) plus $120 (Simple Interest), or $1120. At compound interest the amount due Jan. 1, 1882, would be $1060 ($1000 + $60); the amount due Jan. 1, 1883, would be $1060 plus $63.60 (6% of $1060), or $1123.60. The simple interest for 2 years would be $120; the compound interest for the same time, $123.60. When the word interest is used alone, simple interest is understood.

296. Legal Interest is the interest according to the rate per cent. fixed by law for cases in which the rate per cent. is not specified. By special agreement between parties in certain States, interest may be received at a rate higher than the legal rate. In most of the States, this rate is limited. See Art. **298**.

297. Usury is the taking of a higher rate of interest than that allowed by law. A person taking usury is liable to certain penalties differing in the several States.

298. The following table, prepared from information received from the Secretaries of the several States and Territories, April 1, 1880, shows in the first column the legal rate of interest when no rate is specified in the contract, and in the second column the maximum rate allowed by law.

State or Territory.	Rate.		State or Territory.	Rate.	
Alabama	8%	8%	Mississippi	6%	10%
[a]Alaska (Ter.)	....	....	[e]Missouri	6%	10%
Arkansas	6%	10%	[f]Montana (Ter.)	10%	Any
[b]Arizona (Ter.)	10%	Any	Nebraska	7%	10%
[c]California	10%	Any	Nevada	10%	Any
[d]Colorado	10%	Any	New Hampshire	6%	6%
Connecticut	6%	6%	New Jersey	6%	6%
Dakota (Ter.)	7%	12%	New Mexico (Ter.)	....	Any
Delaware	6%	6%	New York	6%	6%
Florida	8%	Any	North Carolina	6%	8%
Georgia	7%	8%	Ohio	6%	8%
Idaho (Ter.)	10%	18%	Oregon	10%	12%
Illinois	6%	8%	Pennsylvania	6%	6%
Indian (Ter.)	6%	Any	Rhode Island	6%	Any
Indiana	6%	8%	South Carolina	7%	7%
Iowa	6%	10%	Tennessee	6%	6%
Kansas	7%	12%	Texas	8%	12%
Kentucky	6%	6%	Utah (Ter.)	....	Any
Louisiana	5%	8%	Vermont	6%	6%
Maine	6%	Any	Virginia	6%	6%
Maryland	6%	6%	Washington (Ter.)	10%	Any
Massachusetts	6%	Any	West Virginia	6%	8%
Michigan	7%	10%	Wisconsin	7%	10%
Minnesota	7%	10%	Wyoming (Ter.)	12%	Any

(a) Not organized.

(b) "Pawnbrokers are allowed to charge 5% per month."

(c) "On judgments recovered in the courts 7%, but must not be compounded in any manner."

(d) "Most banks pay 6% on time deposits and charge from 1 to 2% per month on loans."

(e) "Parties may contract in writing for the payment of interest upon interest, but it shall not be compounded oftener than once a year."

(f) "The usual rate at which money is loaned is 18%, sometimes 24%."

299. Interest for Parts of a Year.—Although many of the States have rigid laws in regard to the rate per cent. to be charged per annum, few of them specify on what basis interest should be reckoned for a period of time less than a year. The following methods are in common use:

1. Finding the time in months and days (Compound Subtraction, Art. **210**, 1), and regarding the months as twelfths of a year, and the days as thirtieths of a month or 360ths of a year. This method, although implied by the general interest laws * of the State of New York, is not uniform, since it allows the same interest for February with its 28 days as for March with its 31 days. Its results are sometimes greater and sometimes less than those of accurate interest.

2. Finding the exact time in days (**210**, 2) and regarding the days as 360ths of a year. Since a day is $\frac{1}{360}$ of a year, this method produces too great a result. It is however used by merchants, brokers, and bankers generally, and by many banks † in discounting notes. 6% by this method is equivalent to $6\frac{1}{12}$% accurate interest.

3. **Accurate Interest.**—Finding the exact time in days (**210**, 2) and regarding the days as 365ths of a year. This method is used by the United States government, and by some merchants and banks; but, on account of its inconvenience when interest tables are not used, it is not generally adopted.

NOTES.—1. By the first method, the time from July 10 to Sept. 10, would be 2 months, and the interest would be $\frac{2}{12}$ or $\frac{1}{6}$ of the interest for one year. On \$10000 at 6% for 2 months, the interest would be \$100 ($\frac{1}{6}$ of .06 of \$10000).

2. By the second method, the interval between the same dates would be 62 days, and the interest would be $\frac{62}{360}$ of the interest for one year. On \$10000 at 6% for $\frac{62}{360}$ of a year, the interest would be \$103.33 ($\frac{62}{360}$ of .06 of \$10000).

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* "For the purpose of calculating interest, a month shall be considered the twelfth part of a year, and as consisting of thirty days; and interest for any number of days less than a month shall be estimated by the proportion which such number of days shall bear to thirty." (R. S., page 1165.)

† According to the banking laws of the State of New York, banks are authorized in discounting notes to charge interest in advance for the exact number of days which the note has to run (Ch. XVIII, Title 2, § 800).

This law appears to conflict with the law quoted above which implies that the time shall be found in months and days. It does not state whether the days shall be regarded as 360ths or 365ths of a year.

3. By the third method, the interval between the same dates would be 62 days as in the second method, and the interest would be $\frac{62}{365}$ of the interest for one year. On \$10000 at 6% for $\frac{62}{365}$ of a year, the interest would be \$101.92 ($\frac{62}{365}$ of .06 of \$10000).

4. The difference between ordinary interest and accurate interest for the same number of days is $\frac{1}{73}$ of the former, or $\frac{1}{72}$ of the latter ( ). Thus in the above example, the difference between the results, \$1.41 (\$103.33–101.92), is $\frac{1}{73}$ of \$103.33, or $\frac{1}{72}$ of \$101.92.

5. Unless the words "Accurate Interest" are used, all computations in this book are made on the basis of 360 days to the year.

300. Interest is an application of percentage, the element of time being introduced. Therefore the four elements or parts in interest are the Principal (the Base), the Rate, the Interest (the Percentage), and the Time; any three of which being given, the other may be found.

301. To find the interest for any number of years and months.

Ex. What is the interest and amount of \$324, for 2 *yr.* 3 *mo.*, at 8%?

OPERATIONS.

\$324	Principal.	Or	\$324
.08			.18
25.92	Interest for 1 *yr.*		2592
$2\frac{1}{4}$			324
648			58.32
5184			324.
58.32	Interest for $2\frac{1}{4}$ *yr.*		\$382.32
324	Principal.		
\$382.32	Amount for $2\frac{1}{4}$ *yr.*		

ANALYSIS.—At 8%, the interest of \$324 for 1 year is .08 of \$324 (the Principal), or \$25.92. If the interest of \$324 for 1 year at 8% is \$25.92, for 2 *yr.* 3 *mo.* ($2\frac{1}{4}$ *yr.*), it is $2\frac{1}{4}$ times \$25.92, or \$58.32. The amount is \$324 plus \$58.32, or \$382.32.

302. RULE.—*To find the interest, multiply the principal by the rate per cent. expressed decimally, and that product by the number of years, and the months as a fraction of a year.*

To find the amount, add the principal to the interest.

NOTES.—1. When the rate per month is given, apply the same rule, *i. e.*, multiply the principal by the rate per month expressed decimally, and that product by the number of months.

2. Instead of multiplying by the rate and time separately, the process may be shortened by multiplying the principal by the product of the rate and time. In the above example, multiply $324 by .18 ($2\frac{1}{4} \times .08$).

EXAMPLES.

303. Find the interest of

1. $875 for 2 *yr.* at 7%.
2. $642.50 for 3 *yr.* at 6%.
3. $1010.10 for 6 *yr.* 6 *mo.*, at 8%.
4. $3010.75 for 3 *yr.* 4 *mo.*, at 7%.
5. $3745.80 for 4 *yr.* 1 *mo.*, at 6%.
6. $816.40 for 5 *yr.* 3 *mo.*, at 5%.
7. $1275 for 7 *yr.* at 6%.
8. $2789.40 for 3 *yr.* 2 *mo.*, at $4\frac{1}{2}$%.
9. $456.75 for 4 *yr.* 8 *mo.*, at 5%.
10. $10180 for 3 *yr.* 4 *mo.*, at 10%.

NOTE.—In the following examples find the time by Compound Subtraction.

11. What is the interest of $6488 from May 3, 1879, to Sept. 3, 1881, at 7%?

12. What is the amount of $396.60 from Aug. 16, 1880, to Dec. 16, 1882, at 8%?

13. Find the interest of $864.30 from Jan. 1, 1881, to June 1, 1883, at 4%.

14. Compute the interest of $250.75 from Nov. 20, 1882, to July 20, 1884, at $4\frac{1}{2}$%.

15. Loaned on interest, New York, Dec. 16, 1880, $1739.75 (no rate specified); what amount should I receive, June 16, 1881?

16. In settling with a merchant Oct. 3, 1882, I gave my note for $254.60, at 7%; what must be paid Aug. 3, 1883?

304. To find the ordinary interest (360 days to the year) for any rate and time.

305. 60-day Method at 6%.—6% for 12 months or 1 year, is equivalent to 1% for 2 months (60 days), or $\frac{1}{6}$ of one year. 1% of any amount is readily ascertained by placing the point two places to the left. Hence the interest of any sum at 6% per annum for 2 months, or 60 days, may be found by placing the point two places to the left.

NOTE.—It will be found advantageous to use a perpendicular line as a separatrix in solving examples by this method. All necessity for pointing off will then be dispensed with, and confusion prevented.

Ex. What is the interest of \$1236 for 80 *da.*, at 6%?

OPERATION.

\$12	36	= int. for 60 *da.*
4	12	= " " 20 *da.*
\$16	48	= " " 80 *da.*

ANALYSIS.—The interest of \$1236 at 6% for 60 *da.* is found to be \$12.36, by the process already explained. If the interest for 60 *da.* is \$12.36, for 20 *da.* ($\frac{1}{3}$ of 60), it will be $\frac{1}{3}$ of \$12.36, or \$4.12. Hence for 80 *da.*, it will be \$12.36 plus \$4.12, or \$16.48.

Ex. What is the interest of \$864 for 1 *yr.* 10 *mo.* 15 *da.*, at 6%?

OPERATION.

\$8	64	= int. for 60 *da.*
	11	
95	04	= int. for 22 *mo.*
2	16	= " " 15 *da.*
\$97	20	= required int.

ANALYSIS.—The interest of \$864 at 6% for 2 *mo.* is \$8.64. For 1 *yr.* 10 *mo.* (22 *mo.*), it will be 11 times \$8.64, or \$95.04. If the interest for 60 *da.* is \$8.64, for 15 *da.* ($\frac{1}{4}$ of 60), it will be $\frac{1}{4}$ of \$8.64, or \$2.16. Hence the interest for the given time will be \$95.04 plus \$2.16, or \$97.20.

Ex. What is the interest of \$1732.80 for 2 *yr.* 9 *mo.* 23 *da.*, at 7%?

OPERATION.

\$17	32.80	= int. for 2 *mo.* or 60 *da.*
	$16\frac{1}{2}$	
8	6640	
103	9680	
173	280	
5	776	= int. for 20 *da.*
	866	= " " 3 "
6)292	554	= " " given time at 6%.
48	759	= " " " " 1%.
\$341	313	= " " " " 7%.

ANALYSIS.—The interest for 2 *mo.*, forming the basis, is \$17.328. Multiply this by $16\frac{1}{2}$, to find the interest for 33 *mo.* (2 *yr.* 9 *mo.*). As 23 is not an aliquot part of 60, take 20, which is $\frac{1}{3}$ of 60, and 3, which is $\frac{1}{20}$ of 60. Divide the basis, which is the interest for 60 *da.*, by 3, to find the interest for 20 *da.* (\$5.776); and the same sum by 20, to find the interest for 3 *da.* (\$0.866). By adding these various sums, we have the interest for the given time at 6% (\$292.554). To this result add $\frac{1}{6}$ of itself, which is the interest for the given time at 1%, and the required interest is obtained (\$341.31).

306. Aliquot Parts of 60.—$1 = \frac{1}{60}$; $2 = \frac{1}{30}$; $3 = \frac{1}{20}$; $4 = \frac{1}{15}$; $5 = \frac{1}{12}$; $6 = \frac{1}{10}$; $10 = \frac{1}{6}$; $12 = \frac{1}{5}$; $15 = \frac{1}{4}$; $20 = \frac{1}{3}$; $30 = \frac{1}{2}$.

NOTES.—1. To divide by 10, place the figures of the basis one place to the right.

2. To divide by 20, 30, or 60, divide by the first figure and write the quotient figures one place to the right.

307. If the number of days given is other than any of the above, which are aliquot parts of 60, it will need to be so separated that the component parts will be aliquot parts of 60.

Numbers not aliquot parts of 60, with best divisions: $7 = 6+1$; $8 = 6+2$; $9 = 6+3$; $11 = 6+5$, or $10+1$; $13 = 10+3$; $14 = 12+2$; $16 = 10+6$; $17 = 12+5$, or $15+2$; $18 = 12+6$. (The interest for 18 days may be found by multiplying the basis by 3, and placing the figures of the product one place to the right); $19 = 15+4$, or $10+6+3$; $21 = 15+6$; $22 = 20+2$ ($2 = \frac{1}{10}$ of 20); $23 = 20+3$; $24 = 12+12$ (Or multiply by 4 and place the figures of the product one place to the right); $25 = 20+5$ ($5 = \frac{1}{4}$ of 20); $26 = 20+6$; $27 = 12+15$; $28 = 12+12+4$ ($4 = \frac{1}{3}$ of 12), or $20+6+2$; $29 = 12+12+5$, or $20+6+3$.

308. RULE.—*Draw a perpendicular line two places to the left of the decimal point; the result will be the interest at 6% for 2 months, or 60 days, the dollars being on the left, and the cents on the right of this line. Multiply this result by one-half the total number of months. To this product, add that proportion of the interest for 60 days, which the given number of days is of 60.*

309. The interest for any other rate may be found from the interest at 6% as follows: At 1%, divide by 6; at $1\frac{1}{2}$%, divide by 4; at 2%, divide by 3; at 3%, divide by 2; at 4%, subtract $\frac{1}{3}$; at $4\frac{1}{2}$%, subtract $\frac{1}{4}$; at 5%, subtract $\frac{1}{6}$; at 7%, add $\frac{1}{6}$; at 8%, add $\frac{1}{3}$; at 9%, add $\frac{1}{2}$; at 10%, divide by 6, and multiply by 10 by placing the point to the right one place; at 12%, multiply by 2. At any per cent., divide by 6 and multiply by the rate.

310. 6% Method.—At 6%, the interest for one year is .06 of the principal. For one month, $\frac{1}{12}$ of a year, it will be $\frac{1}{12}$ of .06, or .00$\frac{1}{2}$ (.005). For one day, $\frac{1}{30}$ of a month, it will be $\frac{1}{30}$ of .005, or .000$\frac{1}{6}$.

Ex. What is the interest of $864, at 6%, for 2 *yr.* 7 *mo.* 20 *da.*?

OPERATION.

$2 \times .06 = .12$

$7 \times .00\frac{1}{2} = .035$

$20 \times .000\frac{1}{6} = .003\frac{1}{3}$

$.158\frac{1}{3}$

$864

$.158\frac{1}{3}$

288

6912

4320

864

$136.800

ANALYSIS.—If the interest for 1 *yr.* is .06 of the principal, for 2 *yr.* it will be twice .06, or .12. If the interest for 1 *mo.* is $.00\frac{1}{2}$ of the principal, for 7 *mo.* it will be 7 times $.00\frac{1}{2}$, or .035. If the interest for 1 day is $.000\frac{1}{6}$ of the principal, for 20 *da.* it will be 20 times $.000\frac{1}{6}$, or $.003\frac{1}{3}$. Hence the interest for the given time will be $.158\frac{1}{3}$ of the principal ($864), or $136.80.

311. RULE.—*Multiply the given principal by the decimal obtained by taking for every year six hundredths, one-half as many hundredths as there are months, and one-sixth as many thousandths as there are days. The product will be the interest at 6%.*

NOTES.—1. In using this method, to multiply by $\frac{2}{3}$, write $\frac{1}{3}$ twice; to multiply by $\frac{5}{6}$, take $\frac{1}{2}$ and $\frac{1}{3}$.

2. The interest at any other per cent. may be found as in Art. **309**.

3. The decimal obtained by the above rule, if regarded as cents and mills, expresses the interest of $1 for the given time at 6%. The interest of $1 at 6% for 1 year is $.06; for 1 month, $.00$\frac{1}{2}$, or $.005; for 1 day, $.000$\frac{1}{6}$.

312. 6% Method for Days.—This is a modification of the preceding method, and may be applied to any example if the time is reduced to days.

Ex. What is the interest of $1735 for 173 days at 6%?

OPERATION.

$1735

173

5205

12145

1735

6) 300155

$50.025+

ANALYSIS.—The interest of $1735 for 173 days is equivalent to the interest of 173 times $1735, or $300155 for 1 day. Since the interest of $1 for 1 day is $\frac{1}{6}$ of a mill, or $.000\frac{1}{6}$ of the principal, the interest of $300155 for 1 day is as many mills as 6 is contained times in 300155, or 50025 mills, or $50.03.

313. RULE.—*Multiply the principal by the number of days, divide the product by 6, and place the point 3 places to the left. The result will be the interest at 6%.*

NOTES.—1. The interest at any other per cent. may be found as in Art. **309**. To find the interest at 3%, divide by 12 instead of 6; at 4%, by 9; at 9%, by 4.

2. If the principal is a multiple of the divisor (6 in the above example), time can be saved by performing the division first. Thus, to find the interest of $1200 for 113 days, divide 1200 by 6, and multiply the quotient 200 by 113, producing 22600. By pointing off three places, the required interest is $22.60.

EXAMPLES.

314. What is the interest of

1. $375.60 for 8 *mo.* 20 *da.*, at 6%?
2. $1727 for 7 *mo.* 15 *da.*, at 6%?
3. $449.38 for 1 *yr.* 4 *mo.* 12 *da.*, at 6%? At 7%?
4. $285 for 1 *yr.* 5 *mo.* 10 *da.*, at 6%? At 5%?
5. $432.65 for 2 *yr.* 2 *mo.* 6 *da.*, at 6%? At 8%?
6. $1235 for 2 *yr.* 5 *mo.* 5 *da.*, at 6%? At 4%?
7. $445.25 for 5 *mo.* 4 *da.*, at 6%? At 9%?
8. $1000 for 93 days, at 6%? At 7%?
9. $2416.60 for 72 days, at 6%? At 7%?
10. $3210 for 62 days, at 6%? At 8%?
11. $735 for 75 days, at 6%? At 5%?
12. $812.45 for 121 days, at 6%? At 4%?
13. $2440.50 for 97 days, at 6%? At 7%?
14. $3125 for 38 days, at 6%? At 7%?
15. $247.50 for 69 days, at 6%? At 5%?
16. $512.45 for 5 *mo.* 11 *da.*, at 6%? At 7%?
17. $1478 for 1 *yr.* 2 *mo.* 13 *da.*, at 6%? At 8%?
18. $2810.60 for 9 *mo.* 24 *da.*, at 6%? At 5%?
19. $944.50 for 1 *yr.* 10 *mo.* 22 *da.*, at 6%? At $4\frac{1}{2}$%?
20. $575 for 2 *yr.* 8 *mo.* 16 *da.*, at 6%? At 9%?
21. $1112 for 3 *mo.* 14 *da.*, at 6%? At $4\frac{1}{2}$%?
22. $5285 for 1 *yr.* 6 *mo.* 21 *da.*, at 6%? At 3%?
23. $7218 for 11 *mo.* 18 *da.*, at 6%? At 12%?
24. $416.75 for 8 *mo.* 17 *da.*, at 6%? At 7%?
25. $1235 for 2 *yr.* 1 *mo.* 19 *da.*, at 6%? At 8%?
26. $575.60 for 1 *yr.* 4 *mo.* 23 *da.*, at 6%? At 5%?
27. $2214 for 4 *mo.* 25 *da.*, at 6%? At 4%?

28. \$6315 for 5 *mo.* 29 *da.*, at 6%? At 9%?

29. \$4312 for 4 *mo.* 26 *da.*, at 6%? At 4½%?

30. \$384.30 for 2 *mo.* 28 *da.*, at 6%? At 3%?

31. \$1296 for 1 *yr.* 11 *mo.* 27 *da.*, at 6%? At 12%?

32. \$4375 for 2 *yr.* 8 *mo.* 24 *da.*, at 6%? At 5½%?

NOTE.—Find the time in the following examples both in months and days, and in exact days (**210**).

33. \$1234 from May 10 to Dec. 4, at 5%? At 4¾%?

34. \$444.40 from Jan. 13 to Nov. 2, at 4%? At 5¼%?

35. \$575.20 from June 5, 1882, to Feb. 4, 1883, at 7%? At 5%?

36. \$2375 from July 17, 1884, to Nov. 27, 1885, at 6%? At 3½%?

37. \$3212 from Aug. 24, 1881, to Jan. 20, 1884, at 4%? At 4½%?

38. \$475.80 from May 12, 1882, to Feb. 1, 1884, at 7%? At 10%?

39. Find the interest of \$180 for 253 days, at 6%. At 8%.

NOTE.—In many examples, labor can be saved by having the time and principal exchange places. In the above example, the interest of \$180 for 253 days is the same as \$253 for 180 days (\$2.53 × 3).

40. Find the interest of \$600 for 173 days at 9%. At 4%.

41. Find the interest of \$3000 for 111 days at 12%. At 3%.

42. Find the interest of \$1800 from Jan. 17 to Oct. 2, at 6%. At 4¾%.

43. Find the interest of \$540 from May 11 to Dec. 18, at 5%. At 4½%.

44. If \$9200 is loaned Sept. 18, 1882, at 6%, what is due May 9, 1885? (Time by C. S.)

45. What is a banker's gain in 1 year on \$10000 deposited at 6%, and loaned 11 times at 1½% a month?

46. A note for \$1421, with interest after 4 months, at 7%, was given Dec. 1, 1881, and paid Aug. 12, 1883. What was the amount due? (C. S.)

47. Nov. 6, 1881, I bought a lot of grain for \$753.20; Dec. 16, I sold a part of it for \$375.60; and, Dec. 31, I sold the remainder for \$411.40. Money being worth 6%, how much did I gain by the transaction?

ACCURATE INTEREST.

315. To find the accurate interest (365 days to the year) for any rate and time. See Art. **299.**

Ex. What is the accurate interest of $865, at 4%, from June 21 to Dec. 13?

OPERATION.

$865 Principal.
.04
34.60 Interest for 1 yr.
175
365) 6055.00 (16.59

ANALYSIS.—From June 21 to Dec. 13, there are 175 days. The interest of $865 for 1 *yr.*, at 4%, is $34.60. For 175 days, $\frac{175}{365}$ of 1 *yr.*, it is $\frac{175}{365}$ of $34.60 $\left(\frac{175 \times 34.60}{365}\right)$, or $16.59.

316. RULE.—*Multiply the principal by the rate per cent. expressed decimally. The result will be the interest for 1 year.*

Multiply the interest for 1 year by the number of days, and divide the product by 365.

NOTES.—1. When the number of days is a multiple of 5, multiply by $\frac{1}{5}$ the number of days, and divide the product by 73. In the above example, $865 × .04 × 35 ÷ 73 = $16.59.

2. To find the interest at any per cent., multiply by twice the rate as an integer, by the number of days, divide the product by 73, and point off 3 places. In the above example, $865 × 8 × 175 ÷ 73000 = $16.59.

3. To find the interest at 5%, multiply the principal by the number of days, divide the product by 73, and point off 2 places. From this result to find the interest at 6%, add $\frac{1}{6}$; 4½%, subtract $\frac{1}{10}$; 4%, subtract $\frac{1}{8}$.

317. Accurate Interest from Ordinary Interest.—The difference between ordinary interest and accurate interest for 1 day equals the difference between $\frac{1}{360}$ and $\frac{1}{365}$ of a year's interest.

$$\frac{1}{360} - \frac{1}{365} = \frac{365 - 360}{365 \times 360} = \frac{5}{365 \times 360} = \frac{5}{365} \text{ of } \frac{1}{360} = \frac{1}{73} \text{ of } \frac{1}{360}.$$

$$\frac{5}{365 \times 360} = \frac{5}{360} \text{ of } \frac{1}{365} = \frac{1}{72} \text{ of } \frac{1}{365}.$$

The difference between the two methods is $\frac{1}{73}$ of ordinary interest, or $\frac{1}{73}$ of accurate interest (**299,** Note 4). Therefore, from ordinary interest to find accurate interest subtract $\frac{1}{73}$.

In reckoning accurate interest, on account of the many short methods of ordinary interest, many accountants prefer to calculate ordinary interest first, and then make the necessary deduction.

Since $\frac{1}{73}$ is about $1\frac{1}{3}\%$, the following approximate method may be used in reducing ordinary interest to accurate interest: *From the ordinary interest subtract 1% and $\frac{1}{3}\%$ of itself.*

Ex. Reduce \$32.70 ordinary interest to accurate interest.

OPERATION.

32.70	
.327	1%
32.373	
.109	$\frac{1}{3}\%$
32.264	

NOTE.—The exact result should be \$32.252. The results by this method are too great by 1 cent for each \$27 interest; \$.036 for each \$100 interest; \$.36 for each \$1000 interest. Where greater accuracy is required, the necessary correction can be made.

EXAMPLES.

318. What is the accurate interest of

1. \$435.32, at 6%, for 25 days? *5.* \$292, at $3\frac{1}{2}\%$, for 140 days?
2. \$6030, at 5%, for 141 days? *6.* \$438, at 6%, for 210 days?
3. \$780, at 6%, for 90 days? *7.* \$350, at 4%, for 150 days?
4. \$437.80, at 7%, for 63 days? *8.* \$500, at $4\frac{1}{2}\%$, for 100 days?
9. \$3110.45, at $5\frac{1}{2}\%$, for 90 days?
10. \$373.70, at 7%, from June 4 to Dec. 28?
11. \$500, at 6%, from July 24, to Sept. 16?
12. \$365, at 6%, from June 30 to Dec. 21?
13. \$1080, at 5%, from May 9, 1878, to Jan. 30, 1879?
14. \$1728, at 7%, from Jan. 6, 1878, to Jan. 21, 1880?
15. Required the exact interest on three U. S. bonds of \$5000 each, at $3\frac{1}{2}\%$, from July 1 to Aug. 11.
16. What is the interest on three U. S. bonds of \$1000 each, at $4\frac{1}{2}\%$, from Sept. 1 to Nov. 15?
17. What is the interest on a \$5000 U. S. bond, at 4%, from Oct. 1 to Dec. 16?
18. What is the interest on a U. S. bond of \$1000, bearing $3\frac{1}{2}\%$ interest, from May 1 to July 19?
19. What is the interest on a \$500 U. S. bond, at 4%, from Apr. 1 to May 10?
20. What is the interest on a \$5000 U. S. bond from Nov. 1, 1881, to Jan. 3, 1882, at $3\frac{1}{2}\%$?
21. What is the difference between ordinary and accurate interest of \$10000 for 219 days at 6%?

PROBLEMS IN INTEREST.

319. To find the rate, the principal, interest or amount, and time, being given.

Ex. At what rate will \$720 in 1 *yr.* 4 *mo.* 10 *da.*, produce \$44.10 interest?

OPERATION.

\$7	20	
	8	
57	60	
1	20	
6) 58	80	
\$9	80) \$44.10 ($4\frac{1}{2}$ *Ans.*	

ANALYSIS. — The interest on a given principal for a given time is in proportion to the rate per cent. At one per cent., \$720 will in 1 *yr.* 4 *mo.* 10 *da.*, produce \$9.80 interest. To produce \$44.10 interest, the required rate must be as many times 1%, as \$9.80 are contained times in \$44.10, or $4\frac{1}{2}$ times. Hence the answer is $4\frac{1}{2}$%.

320. RULE.—*Divide the given interest by the interest of the given principal, for the given time, at 1%.*

NOTE.—When the amount is given, find the interest by subtracting the principal from the amount.

EXAMPLES.

321. At what rate will

1. \$864 in 8 *mo.* 10 *da.* produce \$42 interest?
2. \$1000 in 9 *mo.* 9 *da.* produce \$54.25 interest?
3. \$852 in 1 *yr.* 7 *mo.* 16 *da.* amount to \$935.21?
4. \$1926 in 2 *yr.* 8 *mo.* 24 *da.* produce \$263.22 interest?
5. \$375.60 in 1 *yr.* 10 *mo.* 22 *da.* amount to \$425.41?
6. \$1872 in 7 *mo.* 17 *da.* produce \$41.31 interest?
7. \$435.60 in 1 *yr.* 2 *mo.* 18 *da.* amount to \$478?
8. \$1338.72 in 6 *mo.* 27 *da.* produce \$34.64 interest?
9. \$1728 in 8 *mo.* 21 *da.* amount to \$1778.11?
10. \$3456 in 5 *mo.* 8 *da.* produce \$91.01 interest?
11. \$5280 in 11 *mo.* 11 *da.* amount to \$5720.12?
12. \$1234 in 8 *mo.* 22 *da.* produce \$80.83 interest?
13. \$6975 in 3 *mo.* 28 *da.* amount to \$7215.06?
14. \$525 in 1 *yr.* 11 *mo.* 18 *da.* produce \$309.75 interest?
15. \$500 in 3 *yr.* 11 *mo.* 12 *da.* amount to \$658?
16. \$4680 in 2 *yr.* 6 *mo.* 11 *da.* produce \$710.58 interest?
17. \$614.45 in 162 days amount to \$633.805?

322. To find the time, the principal, interest or amount, and rate, being given.

Ex. In what time will $426, at 6%, produce $59.427 interest?

OPERATIONS.

```
  $426                                  Or   $426
   .06                                        .06
$25.56 ) $59.427 ( yr. 2.325            $25.56 ) $59.427 ( 2 yr.
         55 12          12                       51 12
          8 307  mo. 3.900                         8.307
          7 668         30                            12
            6390 da. 27.000             $25.56 ) 99.684 ( 3 mo.
            5112                                  76.68
            12780                                 23.004
            12780                                     30
                0                       $25.56 ) 690.120 ( 27 da.
```

ANALYSIS.—The interest on a given principal at a given rate %, is in proportion to the time. In one year, $426, at 6%, will produce $25.56 interest. To produce $59.427 interest, it will require as many years as $25.56 is contained times in $59.427, or 2.325 *yr.* 2.325 *yr.* equal 2 *yr.* 3 *mo.* 27 *da.* (**201**).

323. RULE.—*Divide the given interest by the interest of the given principal, at the given rate, for 1 year.*

The integral part of the quotient will be years. Reduce the decimal, if any, to months and days (**201**).

EXAMPLES.

324. In what time will

1. $3000, at 7%, produce $108.50 interest?
2. $1728, at 6%, amount to $1872?
3. $3932, at 7%, produce $597.88 interest?
4. $735, at 5%, amount to $742.66?
5. $1222.25, at 6%, produce $39.52 interest?
6. $375.60, at 7%, amount to $425.41?
7. $1461.75, at 6%, produce $420.25 interest?
8. $1200, at 3¾%, amount to $1413?
9. $4500, at 5%, produce $181.25 interest?
10. $276.50, at 10%, amount to $303.46?

11. $1020, at 6%, produce $89.25 interest?
12. $6495, at 7%, amount to $7161.81?
13. $100, at 6%, produce $100 interest?
14. $125, at 7%, amount to $375?

325. To find the principal, the interest, time, and rate, being given.

Ex. What principal will produce $152.64 interest, in 1 *yr.* 5 *mo.* 20 *da.*, at 6%?

OPERATION.

$.088\frac{1}{3}$) $152.64 (1728.
3 3
.265) 457.920
265
1929
1855
742
530
2120
2120
0

ANALYSIS.—The interest on any principal is as many times greater than the interest of $1, as that principal is greater than $1. One dollar, in 1 *yr.* 5 *mo.* 20 *da.*, at 6% (**310**), will produce $.088$\frac{1}{3}$ interest. To produce $152.64, the principal must be as many times $1 as $.088$\frac{1}{3}$ is contained times in $152.64, or $1728.

326. RULE.—*Divide the given interest by the interest of $1 for the given time, at the given rate.*

EXAMPLES.

327. What principal will produce
1. $1235 interest, in 1 *yr.* 8 *mo.* 12 *da.*, at 6%?
2. $49.81, in 9 *mo.* 24 *da.*, at 7%?
3. $186.75, in 1 *yr.* 4 *mo.* 20 *da.*, at 6%?
4. $244.44, in 7 *mo.* 18 *da.*, at 5%?
5. $375.60, in 2 *yr.* 4 *mo.* 6 *da.*, at 8%?
6. $54.25, in 3 *mo.* 3 *da.*, at 7%?
7. $387.40, in 2 *yr.* 8 *mo.*, at $4\frac{1}{2}$%?
8. $456, in 93 *da.*, at 6%?
9. $375, in 63 *da.*, at 7%?
10. $1000, in 1 *yr.* 18 *da.*, at 3%?
11. $538.80, in 10 *mo.* 24 *da.*, at 5%.
12. $416.75, in 8 *mo.* 21 *da.*, at 4%?
13. $645.39, in 4 *yr.* 8 *mo.* 10 *da.*, at 4%?

328. To find the principal, the amount, time, and rate, being given.

Ex. What principal will amount to $1880.64, in 1 *yr.* 5 *mo.* 20 *da.*, at 6%?

OPERATION.

$1.088\frac{1}{3}$) $1880.64 (1728.

3 3

3.265) 5641.920

3265

23769

22855

9142

6530

26120

26120

0

ANALYSIS. — The amounts of different principals for the same time and rate %, are to each other as the principals. One dollar, in 1 *yr.* 5 *mo.* 20 *da.*, at 6% will amount to 1.088\frac{1}{3}$. To amount to $1880.64, the principal must be as many times $1 as 1.088\frac{1}{3}$ are contained times in $1880.64, or $1728.

329. RULE.—*Divide the given amount by the amount of $1 for the given time, at the given rate.*

EXAMPLES.

330. What principal will amount to

1. $1272.254, in 6 *mo.* 6 *da.*, at 6%?
2. $5538.72, in 8 *mo.* 12 *da.*, at 7%?
3. $3695.04, in 1 *yr.* 4 *mo.* 18 *da.*, at 5%?
4. $442.71, in 2 *yr.* 2 *mo.* 24 *da.*, at 8%?
5. $14794.31, in 3 *yr.* 3 *mo.* 3 *da.*, at 6%?
6. $1793.38, in 7 *mo.* 17 *da.*, at 6%?
7. $1010.65, in 5 *yr.* 8 *mo.* 6 *da.*, at 7%?
8. $977.75, in 1 *yr.* 10 *mo.* 10 *da.*, at 6%?
9. $1716.75 in 3 *yr.* 4 *mo.* 21 *da.*, at 4%?
10. $2808.08, in 2 *yr.* 8 *mo.* 12 *da.*, at 8%?
11. $4312.22, in 1 *yr.* 2 *mo.* 11 *da.*, at 12%?
12. $6528.49, in 4 *yr.* 7 *mo.* 6 *da.*, at 5%?
13. $1763.02, in 1 *yr.* 2 *mo.* 21 *da.*, at 6%?
14. $2457.28, in 2 *yr.* 5 *mo.* 23 *da.*, at 6%?
15. $5375.34, in 1 *yr.* 6 *mo.* 15 *da.*, at 4%?
16. $3536.87, in 2 *yr.* 7 *mo.* 10 *da.*, at 9%?
17. $4221.50, in 3 *yr.* 10 *mo.* 27 *da.*, at $4\frac{1}{2}$%?

PRESENT WORTH AND TRUE DISCOUNT.

331. The **Present Worth** of a debt due at some future time is its value now. Theoretically, it is such a sum that, if placed at interest to-day for the given time, would amount to the face of the debt.

332. The **True Discount** is the difference between the face of the debt and the present worth.

This subject is an application of the principle illustrated in Art. **328**, the face of the debt being the amount, the present worth the principal, and the true discount the interest.

In actual business, true discount is little used, banks and merchants generally using bank discount (**355**). True discount is the interest on the present worth for the given time, while bank discount is interest on the face of the debt. The difference is therefore equivalent to the interest on the true discount. For discount on bills, etc., when time does not enter in as an element, see Art. **274**.

Ex. Mr. B owes me $212, payable one year from to-day without interest; what is the present worth of the debt, the current rate of interest being 6%?

ANALYSIS.—Since $1 in one year, at 6%, amounts to $1.06, it would require as many dollars to amount to $212, as $1.06 are contained times in $212, or $200. The true discount is $212 − $200, or $12.

333. RULE.—*I. To find the present worth, divide the face of the debt by the amount of $1 for the given time, at the given rate.*

II. To find the true discount, subtract the present worth from the face of the debt.

EXAMPLES.

334. The current rate of interest being 6%, what is the present worth and true discount of

1. $1000, due 2 years hence?
2. $500, due in 2 *yr.* 4 *mo.*?
3. $600, due in 1 *yr.* 7 *mo.*?
4. $800, due in 9 *mo.* 24 *da.*?
5. $325, due in 2 *yr.* 5 *mo.* 12 *da.*?
6. $175, due in 1 *yr.* 4 *mo.* 16 *da.*?
7. $800, due in 5 *yr.* 8 *mo.* 22 *da.*?
8. $900, due in 6 *yr.* 8 *mo.* 14 *da.*?

9. Mr. C. desiring to pay a bill of $1728 4 months before it was due, was allowed a discount equivalent to the interest on the face of the bill for the unexpired time at 6% per annum (bank discount). How much greater was this discount than the true discount?

10. Goods to the amount of $3750 are sold on a credit of 4 months. For how much cash could the merchant afford to sell the same goods, money being worth 10% per annum?

11. If $10000 will be due me May 28, and $8000 May 16, what discount should I make on the two claims Apr. 1, money being worth 8%?

REVIEW EXAMPLES.

335. *1.* What is the interest of $375.60, for 1 *yr.* 10 *mo.* 16 *da.*, at 6%?

2. What is the amount of $1765 for 7 *mo.* 20 *da.*, at 7%?

3. At what rate will $1234, in 2 *yr.* 2 *mo.* 26 *da.*, produce $138.14 interest?

4. In what time will $585, at 6%, produce $67.08 interest?

5. What principal will, in 1 *yr.* 8 *mo.* 14 *da.*, at 6%, produce $176.22 interest?

6. The semi-annual interest on a mortgage at 7% is $350. What is the face of the mortgage?

7. Mr. B. invests $49500 in a business that pays him $594 per month. What annual rate of interest does he receive?

8. Which is the better investment, and what per cent., one of $8400, yielding $336 semi-annually, or one of $15000, producing $1425 annually?

9. May 18th, a speculator bought 1600 bushels of wheat, at $1.50 a bushel. He afterward sold the whole for $2472 cash, his profit being equivalent to 8% per annum on the amount invested. What was the date of the sale?

10. The par value of Mr. A.'s bank stock is $9000, and he receives a semi-annual dividend of $315. What per cent. is the dividend per annum?

11. Mrs. C.'s son is now 16 *yr.* old; how much must she invest for him at 6%, that, on arriving at age, he may have, with simple interest, $25000?

12. What is the interest of $10000 for 2 days, at 6% per annum, and a commission of $\frac{1}{8}$% per day?

13. A gentleman loaned \$15000, at 6%. Jan. 1, 1880, interest and principal together equalled \$20000. When was the money loaned?

14. Find the interest on \$3000, from Mar. 16 to Dec. 4, at 6%, by the following methods (**299**): 1, ordinary interest and compound subtraction; 2, ordinary interest and exact number of days; 3, accurate interest.

15. Oct. 1, 1880, the loans and discounts of the National Banks of the United States amounted to \$1,041,000,000. At 6%, what would be the difference between the ordinary (360 days) and accurate (365 days) interest of this amount for 1 day?

16. How much is paid for the use of \$1000 from Dec. 2 to Dec. 17, accurate interest at 6%, and a commission of $\frac{1}{12}$% per day being charged?

17. 6% per annum accurate interest and a bonus of $\frac{1}{12}$% per day is equivalent to what rate per annum?

18. A man loaned another a sum of money, payable in 5 months, with interest at the rate of 6%, and at the end of that time received \$666.25 in return. How much did he loan?

19. A speculator borrowed \$10925 at 6%, May 16, 1882, with which he purchased flour at \$6.25 per barrel. June 11, 1883, he sold the flour at \$7.50 per barrel, cash. What did he gain by the transaction?

20. B bought 225 *A.* 24 *sq. rd.* of land, Aug. 18, 1882, at \$4 an acre, borrowing the money to pay for it at 5%. He sold the land April 7, 1886, at an advance of \$299.40 on cost. If meanwhile he paid \$46.50 for taxes on the land, did he gain or lose, and how much?

21. A speculator bought 9000 bu. grain at \$1.80 per bushel, Mar. 18, 1875, the money paid for it being borrowed at 5½%. Dec. 12, 1875, he sold $\frac{2}{3}$ of the grain at \$2.00 per bushel, and the remainder at \$1.90 per bushel. What was gained or lost by the transaction?

22. A person buying a building lot for \$5400, agreed to pay for it in four equal semi-annual installments, with interest at 6%; what was the total amount of money paid, the first payment being made at the time of the purchase?

23. A bill of goods amounting to \$4316.75 is due May 27; how much would settle it May 1 at 6%? How much July 3?

24. A owes B £260 9*s.* 6*d.*, with interest at 5%, for 143 days. He pays 25% of the amount due; how much remains?

NOTE.—In England, interest is usually computed on the basis of 365 days to the year, when the time is given in days. The legal rate in England is 5%. To calculate interest on English money, reduce the shillings and pence to the decimal of a pound (see Art. **204**, Ex. 7, Note), apply any of the methods under Art. **316**, and reduce the resulting decimal to shillings and pence.

Find the accurate interest of

25. £425, from Aug. 4 to Dec. 28, at 5%.

26. £625 12*s.*, from Jan. 12 to Apr. 1, at 4%.

27. £717 16*s.* 10*d.*, from Mar. 3 to June 16, at 4½%.*

28. £429 10*s.* 8*d.*, from Sept. 16 to Nov. 30, at 3%.

29. £516 18*s.* 3*d.*, from Aug. 1 to Oct. 18, at 3½%.

30. £612 6*s.* 11*d.*, from July 1 to Nov. 3, at 5%.

31. £225 15*s.* 5*d.*, from Feb. 11 to Sept. 8, at 2½%.

32. A commission merchant sold 24160 pounds of leather at 26¾ cents a pound, paid transportation $60.40, cartage $20, his commission being 2½%, and his charge for inspection $20. What were the net proceeds?

33. What per cent. profit does a merchant make who buys at a discount of 20, 10, and 12½%, and sells at the list price?

34. At what per cent. above cost must goods be marked, so that when sold at a discount of 5%, there would be a profit of 25%?

35. A buys a bill of goods amounting to $2776.40, on the following terms:—"4 months, or less 5% cash." He accepts the latter, and borrows the money at 6% to pay the bill. How much does he gain?

36. I purchase books at $2 each less 33⅓%, and 5% for cash. What was the net cost, and what per cent. discount may be given on the list price to produce a net profit of 10%?

37. C of New York sells for D of Atlanta, a quantity of cotton, amounting to $7317.83, and charges a commission of 2½%. By instructions, he invests the proceeds in dry goods, after deducting a commission of 1½% of the amount expended. What was the total commission?

38. A lawyer collected 75% of an account of $3416, charging 5% commission. What amount should he pay over?

* When the time is less than 1 year, and the rate is 6% or less, reject the pence, if less than 6; add 1 shilling, if more than 6. The result will be sufficiently accurate.

ANNUAL INTEREST.

336. When a note contains the words "with interest annually," the laws of New Hampshire and Vermont, if the interest is not paid when due, allow simple interest on the annual interests from the time they become due to the time of payment.

ILLUSTRATION.—A agrees to pay B $6000 in three years from Jan. 1, 1880, with interest annually at 6%. By this contract, $360 becomes due Jan. 1, 1881, and on the first day of January in each year thereafter, until paid; this is the "annual interest." Suppose A does not pay any portion of this interest until Jan. 1, 1883, when the principal becomes due; then A, having had the use of money that his contract required him to pay to B, and B having been deprived of its use, B is entitled to have simple interest added to the annual interest, from the time when the same became due to Jan. 1, 1883; so that on Jan. 1, 1883, B would be entitled to the following sums as interest:

First year's int.	$360 + 2 *yrs.* simple int. thereon,	$43.20 =	$403.20
Second " "	360 + 1 " " " "	21.60 =	381.60
Third " "	360 + 0 (paid when due)	00 =	360
	$1080	$64.80 =	$1144.80

Amount of annual interest	$1080.00
Amount of simple interest accrued upon annual interest .	64.80
Total amount of interest due	$1144.80

In calculating the simple int. upon the annual int., shorten the operation by finding the int. upon the annual int. for the sum of the several periods.

Ex. What is the amount due on the following note July 1, 1885?

$10000. CONCORD, N. H., *January 1, 1882.*

Three years after date, for value received, I promise to pay A. B. THOMPSON, or order, Ten Thousand Dollars, with interest payable annually.

C. A. DOWNS.

OPERATION.

Face of note, on interest from Jan. 1, 1882	$10000.00
Interest from Jan. 1, 1882, to July 1, 1885, 3 *yr.* 6 *mo.*	2100.00
3 items of annual interest ($600 each) are unpaid:	
1st from Jan. 1, 1883, to July 1, 1885, 2 *yr.* 6 *mo.*	
2nd from Jan. 1, 1884, to July 1, 1885, 1 *yr.* 6 *mo.*	
3rd from Jan. 1, 1885, to July 1, 1885, 6 *mo.*	
Int. on the annual int. = int. on $600 for 4 *yr.* 6 *mo.*	162.00
Total amount due July 1, 1885	$12262.00

337. RULE.—*To the given principal and its interest to the date of settlement, add the interest on each annual interest from the time it is due to the date of settlement. The sum will be the amount due at annual interest.*

EXAMPLES.

338. *1.* At 6%, interest payable annually, how much would be due Oct. 1, 1884, according to the laws of New Hampshire, on a note of $8000, dated June 1, 1881, no payments having been made?

2. What amount would be due Jan. 1, 1886, at 6%, on a note for $4200, dated Concord, N. H., May 16, 1882, interest payable annually, and no payments having been made?

3. A note for $10000 was dated Apr. 1, 1882, and payable four years from date without interest. Attached to this note were 8 notes of $400 each for the semi-annual interest due Oct. 1, 1882, Apr. 1, 1883, Oct. 1, 1883, Apr. 1, 1884, Oct. 1, 1884, Apr. 1, 1885, Oct. 1, 1885, Apr. 1, 1886. How much was due, at 8%, Apr. 1, 1886, nothing having been paid?

NOTE.—It is the custom of certain corporations when making loans for long periods of time on collateral security or on bond and mortgage, to have a note or mortgage given without interest for the principal, and to have separate notes given for each sum of annual, semi-annual, or quarterly interest, due and maturing at the time the interest is payable. These notes draw interest after maturity like any other note, and may be collected without disturbing the original loan.

4. What amount would be due July 1, 1884, on a note of $5000, dated July 1, 1882, given for 2 years, with notes for quarterly interest, no payments having been made?

5. Required the amount due Jan. 1, 1883, on a note of $3600, dated June 1, 1881, due in two years, notes for semi-annual interest from date, at 6%, having been given, and nothing having been paid.

6. Find the amount of $1200, at 6%, interest payable annually, from June 16, 1882, to Dec. 28, 1886, no interest having been paid except for the first year.

7. What must be paid, Oct. 16, 1885, in settlement of a note for $2500, dated Manchester, N. H., May 6, 1880, said note promising interest annually, and no interest having been paid?

COMPOUND INTEREST.

339. Compound Interest is interest not only on the principal, but on the interest also after it becomes due (**295**).

1. Interest may be compounded annually, semi-annually, quarterly, etc.

2. Interest upon interest due, or compound interest, cannot be collected by law, that is, payment cannot be enforced; but such a payment is equitable, and the receiver of it, if the debtor is willing or can be induced to pay it, does not constitute usury in the legal sense of the word. In the State of Missouri, parties may contract in writing for the payment of interest upon interest, but it shall not be compounded oftener than once a year (**298**, *f.*).

Ex. What is the compound interest of $1000 for 3 years, at 6%?

OPERATIONS.

$1000.00	Principal.	Or	$1000
60.00	Interest for 1 *yr.*		1.06
1060	Amount for 1 *yr.*, or 2d principal.		1060
63.60	Interest of $1060 for 1 *yr.*		1.06
1123.60	Amount for 2 *yr.*, or 3d principal.		1123.60
67.416	Interest of $1123.60 for 1 *yr.*		1.06
1191.016	Amount for 3 *yr.*		1191.016
1000	Original principal.		1000
191.016	Compound interest for 3 *yr.*		191.016

340. RULE.—*Find the amount of the given principal for the first period of time, and make it the principal for the second. Find the amount of the second principal for the second period of time, and make it the principal for the third; and so continue for the whole time. The last amount is the amount required.*

The last amount, less the given principal, will be the compound interest.

NOTES.—1. When the time is not a multiple of the interest period, find the amount of the principal to the end of the last period; then compute the simple interest on this amount for the remaining time, and add it to the last amount. The sum will be the required amount.

2. The work of computing compound interest may be shortened by using the tables on pages 140 and 141.

341. Table showing the sum to which $1 will increase, at compound interest, in any number of years not exceeding 45.

Yrs.	2%.	2½%.	3%.	3½%.	4%.	4½%.	5%.	6%.	7%.	Yrs.
1	1.0200	1.0250	1.0300	1.0350	1.0400	1.0450	1.0500	1.0600	1.0700	1
2	1.0404	1.0506	1.0609	1.0712	1.0816	1.0920	1.1025	1.1236	1.1449	2
3	1.0612	1.0769	1.0927	1.1087	1.1249	1.1412	1.1576	1.1910	1.2250	3
4	1.0824	1.1038	1.1255	1.1475	1.1699	1 1925	1.2155	1.2625	1.3108	4
5	1.1041	1.1314	1.1593	1.1877	1.2167	1.2462	1.2763	1.3382	1.4026	5
6	1.1262	1.1597	1.1941	1.2293	1.2653	1.3023	1.3401	1.4185	1.5007	6
7	1.1487	1.1887	1.2299	1.2723	1.3159	1.3609	1.4071	1.5036	1.6058	7
8	1.1717	1.2184	1.2668	1.3168	1.3686	1.4221	1.4775	1.5938	1.7182	8
9	1.1950	1.2489	1.3048	1.3629	1.4233	1.4861	1.5513	1.6895	1.8385	9
10	1.2190	1.2801	1.3439	1.4106	1.4802	1.5530	1.6289	1.7908	1.9672	10
11	1.2434	1.3121	1.3842	1.4600	1.5395	1.6229	1.7103	1.8983	2.1049	11
12	1.2682	1.3449	1.4258	1.5111	1.6010	1.6959	1.7956	2.0122	2.2522	12
13	1.2936	1.3785	1.4685	1.5640	1.6651	1.7722	1.8856	2.1329	2.4098	13
14	1.3195	1.4130	1.5126	1.6187	1.7317	1.8519	1.9799	2.2609	2.5785	14
15	1.3459	1.4483	1.5580	1.6753	1.8009	1.9353	2.0789	2 3966	2·7590	15
16	1.3728	1.4845	1.6047	1.7340	1.8730	2.0224	2.1829	2.5404	2.9522	16
17	1.4002	1.5216	1.6528	1.7947	1.9479	2.1134	2.2920	2.6928	3.1588	17
18	1.4282	1.5597	1.7024	1.8575	2.0258	2.2085	2.4066	2.8543	3,3799	18
19	1.4568	1.5987	1.7535	1.9225	2.1068	2.3079	2.5270	3.0256	3.6165	19
20	1.4859	1.6386	1.8061	1.9898	2.1911	2.4117	2.6533	3.2071	3.8697	20
21	1.5157	1.6796	1.8603	2 0594	2.2788	2.5202	2.7860	3.3996	4.1406	21
22	1.5460	1.7216	1.9161	2.1315	2.3699	2.6337	2.9253	3.6035	4.4304	22
23	1.5769	1.7646	1.9736	2.2061	2.4647	2.7522	3.0715	3.8197	4.7405	23
24	1.6084	1.8087	2.0328	2.2833	2.5633	2.8760	3.2251	4.0489	5.0724	24
25	1.6406	1 8539	2.0938	2.3632	2.6658	3.0054	3.3864	4.2919	5.4274	25
26	1.6734	1.9003	2.1566	2.4460	2.7725	3.1407	3.5557	4.5494	5.8074	26
27	1.7069	1.9478	2.2213	2.5316	2.8834	3.2820	3.7335	4.8223	6.2159	27
28	1.7410	1.9965	2.2879	2.6202	2.9987	3.4297	3.9201	5.1117	6.6488	28
29	1.7758	2.0464	2.3566	2.7119	3.1187	3.5840	4.1161	5.4184	7.1143	29
30	1.8114	2.0976	2.4273	2.8068	3.2434	3.7453	4.3219	5.7435	7.6123	30
31	1 8476	2.1500	2.5001	2.9050	3.3731	3.9139	4.5380	6.0881	8.1451	31
32	1.8845	2.2038	2.5751	3.0067	3.5081	4.0900	4.7649	6.4534	8.7153	32
33	1.9222	2.2589	2.6523	3.1119	3.6484	4.2740	5.0031	6.8406	9.3253	33
34	1.9607	2.3153	2.7319	3.2209	3.7943	4.4664	5.2533	7.2510	9.9781	34
35	1.9999	2.3732	2 8139	3.3336	3.9461	4.6673	5.5160	7.6861	10.6766	35
36	2.0399	2 4325	2 8983	3.4503	4.1039	4.8774	5.7918	8.1473	11.4239	36
37	2.0807	2.4933	2.9852	3.5710	4.2681	5.0969	6.0814	8.6361	12.2236	37
38	2.1223	2.5557	3.0748	3.6960	4.4388	5.3262	6.3855	9.1543	13.0793	38
39	2.1647	2.6195	3.1670	3.8254	4.6164	5.5659	6.7048	9.7035	13.9948	39
40	2.2080	2.6851	3.2620	3.9593	4.8010	5.8164	7.0400	10.2857	14.9745	40
41	2.2522	2.7522	3.3599	4.0978	4.9931	6.0781	7.3920	10.9029	16.0227	41
42	2.2972	2.8210	3.4607	4.2413	5.1928	6.3516	7.7616	11.5570	17.1443	42
43	2.3432	2.8915	3.5645	4.3897	5.4005	6.6374	8.1497	12.2505	18.3444	43
44	2.3901	2.9638	3.6715	4.5433	5.6165	6.9361	8.5572	12.9855	19.6285	44
45	2.4379	3.0379	3.7816	4.7024	5.8412	7.2482	8.9850	13.7646	21.0025	45

To find the sum to which a given amount will increase, at compound interest, at any of the rates per cent. and number of years expressed in the above Table:

Multiply the given amount by the sum to which one dollar will increase at the rate and for the number of years required, marking off as many decimals from the product as there are decimals in the multiplier and multiplicand.

NOTES.—1. The amount for any number of years not given in the table may be computed by finding the product for any two numbers of years whose sum equals the given time. Thus, the compound amount of $1 at 6% for 55 years, may be found by multiplying $18.7646, the amount for 45 years, by 1.7908, the amount for 10 years.

2. If the interest is compounded semi-annually, to find the amount from the table, take twice the number of years at one-half the rate. Thus, the amount at 8%, compounded semi-annually, for 5 years, is equivalent to the amount for 10 periods of 6 months each, at 4% for each period, and is the same as the amount for 10 years at 4%. If the interest is compounded quarterly, take 4 times the number of years at one-fourth the rate.

3. The compound interest of $1 is $1 less than the amounts in the above table.

342. Table showing the sum to which $1, paid at the beginning of each year will increase at compound interest, in any number of years not exceeding 50.

Yrs.	3%.	3½%.	4%.	5%.	6%.	7%.	8%.	10%.	Yrs.
1	1.0300	1.0350	1.0400	1.0500	1.0600	1.0700	1 0800	1.1000	1
2	2.0909	2.1062	2.1216	2.1525	2.1835	2.2149	2 2464	2.3100	2
3	3.1836	3.2149	3.2465	3.3101	3.3746	3.4399	3.5061	3.6410	3
4	4.3091	4.3625	4.4163	4.5256	4.6371	4.7507	4.8666	5.1051	4
5	5.4634	5.5502	5.6330	5.8019	5.9753	6.1533	6.3359	6.7156	5
6	6.6625	6.7791	6.8933	7.1420	7.3938	7.6540	7.9228	8.4872	6
7	7.8923	8.0517	8.2142	8.5491	8.8975	9.2598	9.6366	10.4959	7
8	9.1591	9.3685	9.5828	10.0266	10.4913	10.9780	11.4876	12.5795	8
9	10.4639	10 7314	11.0061	11.5779	12.1808	12.8164	13.4966	14.9374	9
10	11.8078	12.1420	12.4864	13.2068	13.9716	14.7836	15.6455	17.5312	10
11	13.1920	13.6020	14 0258	14.9171	15.8699	16.8885	17.9771	20.3843	11
12	14.6178	15 1130	15.6268	16.7130	17.8321	19.1406	20.4952	23.5227	12
13	16.0863	16.6770	17.2919	18.5986	20.0151	21.5505	23.2149	26.9750	13
14	17.5989	18.2957	19.0236	20.5786	22.2760	24.1290	26.1521	30.7725	14
15	19.1569	19.9710	20.8245	22.6575	24.6705	26.8881	29.3243	34.9497	15
16	20.7616	21.7050	22.6975	24.8404	27.2129	29.8402	32.7502	39,5447	16
17	22.4144	23.4997	24.6454	27.1324	29.9057	32.9990	36.4502	44.5992	17
18	24.1169	25.3573	26 6712	29.5390	32.7600	36.3790	40.4463	50.1591	18
19	25.8704	27.2797	28.7781	32.0659	35.7856	39.9955	44.7620	56.2750	19
20	27.6765	29.2635	30.9692	34 7193	38.9927	43.8652	49.4229	63.0025	20
21	29.5368	31.3290	33.2480	37.5052	42.3923	48.0058	54.4568	70.4027	21
22	31.4529	33.4634	35.6179	40.4305	45.9958	52.4861	59.8062	78 5430	22
23	33.4265	35 6665	38.0826	43.5020	49.8156	57.1767	65.7648	87.4978	23
24	35.4593	37.9499	40 6459	46.7271	53.8645	62.2490	72.1059	97.3471	24
25	37.5530	40.3131	43.3117	50.1135	58.1564	67.6765	78.9544	108.1818	25
26	39.7096	42.7591	46.0342	53.6931	62.7058	73.4838	86.3508	120.0999	26
27	41.9309	45.2906	48.9376	57.4026	67.5281	79.6977	94.3388	133.2099	27
28	44.2188	47.9103	51.9663	61.3227	72.6398	86.3465	102.9659	147.6809	28
29	46.5754	50.6227	55.0849	65.4388	78.0582	93.4608	112.2832	163.4940	29
30	49.0027	53-4295	58.3283	69.7603	83.8017	101.0730	122.3459	180.9434	30
31	51.5028	56.3345	61.7015	74.2983	89 8898	109.2182	133.2135	200.1378	31
32	54.0778	59.3412	65.2095	79.0638	96.3432	117.9334	144.9506	221.2515	32
33	56.7302	62.4532	68.8579	84.0670	103.1838	127.2588	157.6267	244.4767	33
34	59.4621	65.6740	72.6522	89.3203	110.4348	137.2369	171.3168	270.0244	34
35	62.2719	69.0076	76.5983	94.8363	118.1209	147.9135	186.1021	298.1268	35
36	65.1742	72.4579	80 7022	100.6281	126.2681	159.3374	202.0703	329.0395	36
37	68.1594	76.0289	84.9703	106.7095	134.9042	171.5610	219.3159	363.0434	37
38	71.2342	79.7249	89.4091	113.0950	144.0585	184.6403	237.5412	400.4478	38
39	74.4013	83.5503	94.0255	119.7998	153.7620	198.6351	258.0565	441.5926	39
40	77.6633	87.5095	98.8265	126.8398	164.0477	213.6096	279.7810	486.8518	40
41	81.0232	91.6074	103.8196	134.2318	174.9506	229.6322	303.2435	536.6370	41
42	84.4839	95.8486	109.0124	141.9933	186.5076	246.7765	328.5830	591.4007	42
43	88.0484	100.2383	114.4129	150.1430	198.7580	265.1208	355.9496	651.6408	43
44	91.7199	104.7817	120.0294	158.7002	211 7425	284.7493	385.5056	717.9048	44
45	95.5015	109 4840	125.8706	167.6852	225.5081	305.7518	417.4261	790.7953	45
46	99.3965	114.3510	131.9454	177.1194	240.0986	328.2244	451.9002	870.9749	46
47	103.4084	119.3883	138.2632	187.0254	255.5645	352.2701	489.1322	959.1723	47
48	107.5406	124.6013	144.8337	197.4267	271.9584	377.9990	529.3427	1056·1896	48
49	111 7969	129.9979	151.6671	208.3480	289.3359	405.5289	572.7702	1162.9085	49
50	116.1807	135.5828	158.7738	219.8154	307.7561	434.9859	619.6718	1280.2993	50

To find the sum to which a given amount, per annum, will increase at compound interest, at any of the rates per cent. and number of years expressed in the above Table:

Multiply the given amount, per annum, by the sum to which one dollar per annum will increase at the rate and for the number of years required, marking off as many decimals from the product as there are decimals in the multiplier and multiplicand.

NOTE.—If the amount be payable semi-annually, and compound interest is to be allowed semi-annually, take the amount for double the number of years at one-half the rate per cent. Thus, for a semi-annual payment of $1 for 10 years at 10 per cent., take the amount of $1 for 20 years at 5 per cent. = $34.7198. For a quarterly payment, take the amount for four times the number of years at one-fourth the rate per cent.

EXAMPLES.

343. *1.* What will $450 amount to at compound interest, in 4 years, compounded annually at 4%? At 3%?

2. Find the compound interest of $360, for 2 years, interest compounded semi-annually at 6%. At 5%.

3. What is the compound interest of $800 for 1 *yr.* 3 *mo.* at 8%, interest compounded quarterly?

4. At compound interest, what is the amount of $1728 for 3 *yr.* 4 *mo.* 16 *da.*, interest compounded annually at 3%? At 6%?

NOTE.—First find the amount for 3 years, and use this amount as the principal for the remaining time.

5. B holds a mortgage against A's property dated Apr. 1, 1881, for $20000, interest payable annually at 6%. The interest due Apr. 1, 1882, is not paid until May 26, 1882. How much is then due, A having consented to pay interest upon interest? (See Note 2, Art. **339**).

NOTE.—In solving the following examples, use the tables in Art. **341-342**.

6. A gentleman deposits in a savings bank $100 when his child is one year old. How much will this amount to when he is 21 years old, interest being compounded semi-annually at 4%? At 5%?

7. If at the age of 25 years, a person places $2000 on interest, compounded annually at 6%, what will be the amount due him when he is 50 years old?

8. What will $625 amount to at compound interest, in 36 years, compounded annually at 3%? At 4%?

9. At the age of 20, and every year thereafter, a young man places $200 at compound interest at 6%. How much will he have at the age of 30? At the age of 40? (See Art. **342**.)

10. How much will a gentleman have at the end of three years, if he places at compound interest at 5%, $300 at the beginning of each year?

11. Mr. B., whose life is insured for $4000, pays an annual premium of $114. How much would this amount to at 6% compound interest in 20 years?

12. A lady deposits $50 in a savings bank Jan. 1 and July 1, of each year; how much will be placed to her credit in 15 years, money being worth 5%, compound interest?

13. What sum must be placed at compound interest, at 6%, to amount to $1000 in 5 years?

NOTE.—In compound interest, as in simple interest, the amounts are proportional to the principals; hence the amount of any principal is as many times greater than the amount of $1, as that principal is greater than $1.

To find the principal, divide the given amount by the amount of $1 for the given time and rate.

In simple interest, the interest on a given principal for a given time is in proportion to the rate per cent., and at a given rate, in proportion to the time; but, in compound interest, such is not the case. If the rate or time be doubled, the interest is more than doubled.

14. How much should a gentleman invest at compound interest, 6%, for his son who is now 6 years old, so that, when he becomes 21 years of age, he may have $10000?

15. In the above example, how much should be invested at the beginning of each year to produce the same sum?

16. A gentleman at his death left $7350 for the benefit of his only son, 12 years old, the money to be paid to him when he should be 21 years of age. How much did he receive, interest at 6%, compounded semi-annually?

17. How much must a person at the age of 25 years, place at compound interest at 6%, so that the amount due him, when he is 50 years old, will be $20000?

18. In the above example, how much should he invest annually to produce the same sum?

COMMERCIAL PAPER.

344. Commercial Paper embraces notes, drafts, bills of exchange, etc.

345. A Note (also called a Promissory Note) is a written promise to pay a certain sum of money on demand or at a specified time.

346. The **Maker** of a note is the person who signs it, and thus becomes responsible for its payment. The **Payee** is the person to whom, or to whose order, it is made payable. The **Face** of a note is the sum promised.

In Note 1, Art. **352**, Peter Cooper is the maker; George Peabody is the payee; the face of the note is $1000.

347. A **Negotiable Note** is a note which is made payable to bearer or to the order of some person (See Notes, Art. **352**).

1. A note is *non-negotiable* when it is payable only to the party named in the note.

2. A negotiable note made in New Jersey must contain the words "without defalcation or discount;" in Missouri, the words "negotiable and payable without defalcation or discount."

3. Negotiable notes payable to order may be sold or transferred by the payee writing his name upon the back of the note. He then becomes an indorser.

348. The **Indorser** of a note or draft is the person who writes his name on the back of it, and by so doing guarantees its payment.

If Mr. Erastus Corning desires to sell or transfer Note 3, Art. **352**, it will be necessary for him to indorse it. If he writes his name only, it is called an indorsement *in blank*, and the note is then payable without further indorsement to any person lawfully holding the same. He may indorse it *in full* by making it payable to a particular person, thus—"Pay to the order of Henry R. Pierson, Erastus Corning." Before it can be again transferred, it will require the indorsement of Henry R. Pierson. For greater security, checks, notes, drafts, etc., are indorsed in full when sent by mail.

If an indorser does not wish to guarantee the payment of a note, draft, etc., he writes "Without recourse" over his name at the time of the indorsement.

Sometimes notes and drafts are drawn to the order of the maker or the drawer (to the order of myself or ourselves) to facilitate their transfer without the indorsement of the holder.

349. A **Draft**, or **Bill of Exchange** is an order or request addressed by one person to another directing the payment of a specified sum of money to a third person or to his order.

350. The **Drawer** of the draft is the person who signs it. The **Drawee** is the person on whom it is drawn. The **Payee** is the person to whom, or to whose order, it is made payable.

In Draft 5, Art. **352**, C. P. Huntington is the drawer; Drexel, Morgan & Co. are the drawees; J. & W. Seligman & Co. are the payees.

1. The person in whose favor the bill is drawn is sometimes called the buyer, and becomes the "remitter." After the bill is presented and accepted, the drawee is called the acceptor, and the draft, an acceptance. The draft then has the same legal significance as a promissory note.

2. A person accepts or promises to pay a draft by writing the word "Accepted" and the date over his name across its face.

3. Drafts are sometimes accepted in the following form :—"Accepted August 20, 1881, and payable at the National Park Bank, New York, G. B. Horton & Co."

4. In the State of New York, both by law and custom, the drawee of a draft may demand 24 hours consideration from the time the draft is presented for acceptance.

When accepted, it must bear the date when first seen by him.

5. To "honor" a draft is to accept it or pay it on being presented.

351. A **Protest** is a formal statement made by a Notary Public, declaring that a draft or note has been presented for payment or acceptance, and was refused.

352. Forms of Notes and Drafts.

1. Demand Note.

$1000. NEW YORK, *August 19, 1881.*

On demand, I promise to pay GEORGE PEABODY, or bearer, One Thousand Dollars. Value received.

PETER COOPER.

The above note is payable on demand,—that is, whenever presented ; is negotiable (payable to bearer) ; and bears interest from date at the legal rate of the State in which it is made. If the words "or bearer" were omitted the note would not be negotiable.

2. Time Note—Interest-bearing.

875\frac{44}{100}$. CINCINNATI, OHIO, *July 16, 1882.*

Six months after date, I promise to pay GEO. C. MILLER, or order, Eight Hundred Seventy-five and $\frac{44}{100}$ Dollars, with interest at eight per cent. Value received.

ALEX. MCDONALD.

The above note is payable 6 *mo.* 3 *da.* after its date, or Jan. 19, 1883 ; is negotiable (payable to order) ; and draws interest from its date at 8% per annum. If the rate of interest was omitted, it would bear interest at the legal rate of the State for such cases, 6%. (See Art. **298**.)

3. Time Note—Without Interest—Payable at a Bank.

$6000. ALBANY, N. Y., *December 4, 1881.*

Sixty days after date, I promise to pay to the order of ERASTUS CORNING, Six Thousand Dollars, at the Second National Bank. Value received.

E. C. KOONZ.

The preceding note is payable 63 days from Dec. 4, 1881, or Feb. 5, 1882. It is payable at the Second National Bank. No interest will be due at maturity (Feb. 5). If the note is not paid at maturity, it will bear interest from that date.

4. Joint and Several Notes.

416\frac{32}{100}$. Worcester, Mass, *May 27, 1882.*

Four months after date, we jointly and severally promise to pay John S. Ballard & Co., or order, Four Hundred Sixteen $\frac{32}{100}$ dollars, with interest from date, value received.

T. K. Earle.
Chas. W. Smith.

If the above note were written "we jointly promise, etc.," it would be called a *joint note*. The makers of a joint note must be sued jointly, each being responsible for one-half of the amount of the note. The makers of a joint and several note may be sued separately, either being responsible for the full amount of the note.

5. Sight Draft.

$8000. San Francisco, Cal., *May 1, 1882.*

At sight, pay to the order of J. & W. Seligman & Co., Eight Thousand Dollars, value received.

C. P. Huntington.

To Drexel, Morgan & Co., New York.

6. Time Draft.

$5000. Burlington, Iowa, *June 18, 1881.*

At sixty days' sight, pay to the order of Addison Ballard, Five Thousand Dollars, value received, and charge to account of

A. G. Adams.

To Barton & Jones, Chicago, Ill.

Drafts are sometimes drawn a certain number of "days after date." For Foreign Bills of Exchange, see Art.

Notes.—1. A note should contain the words "Value received," as a contract without a consideration is not legally binding.

2. Negotiable securities are good in the hands of one who purchases in good faith and before maturity, although the seller may have found or stolen them.

3. Where no place of payment is specified, a promissory note is payable at the maker's place of business, or if none is known, at the residence of the maker.

4. A note or draft must be presented at the place where it is made payable. If at a bank, during banking hours; if at a place of business, during business hours; if at a residence, during family hours; and if the maker, or some one for him, is not ready with legal tender currency to pay it, the holder need not call again. A check, even if certified, is not a legal tender, and may be lawfully refused.

353. Days of Grace and Maturity.—**The day of maturity** is the day on which a note becomes legally due. According to the laws of most of the States, a note is not legally due until three days after the expiration of the time specified in the note, except the note contain the words "without grace." These days are called *days of grace*, but they are of no advantage to the payer, since interest is charged for them as for any others.

1. California has abolished days of grace altogether. In Georgia, Alabama, and Kentucky, grace is allowed on promissory notes only in case they are made payable, or are discounted or left for collection at a bank or private banker's. (March, 1879.)

2. By statute in the State of New York and most of the States, all bills and notes due on Sunday are payable on Saturday, and all due on a legal holiday are made payable on the business or secular day next preceding. Thus, if a holiday falls on Thursday, all notes, etc., must be paid on Wednesday; if a holiday falls on Monday, all notes due Sunday or Monday would be payable on Saturday; if a holiday falls on Saturday, notes due Saturday or Sunday would be payable on Friday.

3. The legal holidays in the State of New York are New Year's Day (Jan. 1), Washington's Birthday (Feb. 22), Decoration Day (May 30), Independence Day (July 4), Election Day (the first Tuesday after the first Monday in November), Thanksgiving Day (the day appointed by the President of the United States and Governor of the State, usually the last Thursday of November), and Christmas (Dec. 25).

4. When a legal holiday falls on Sunday, Monday is, by the statute of New York, made a legal holiday, and notes, etc., maturing on Sunday or Monday, must be paid on the preceding Saturday.

5. A note made due at a fixed date in the future, carries 3 days' grace (unless the words "without grace" are used in the contract). Thus, a note stating that "on May 1, 1882, I promise, etc.," would carry 3 days' grace, and would be payable May 4, 1882.

6. When the time of a note is expressed in months, calendar months are used to determine the day of maturity; when in days, the exact number of days is used.

Thus, a note dated July 16, and payable two months from date, would nominally mature Sept. 16, and, including the three days of grace, would legally mature Sept. 19. A note having the same date, and payable

sixty days from date, would nominally mature Sept. 14, and, including the three days of grace, would legally mature Sept. 17.

7. A note due in one or more months from date, matures on the corresponding day of the month up to which it is reckoned, *if there are so many days in that month;* but if not so many, it then matures on the last day of said month, to which the usual grace must be added. Thus, notes dated Jan. 28, 29, 30, or 31, and payable one month from date, would become due Mar. 3 (Feb. 28 with 3 days' grace added).

8. When drafts are payable a certain time after *sight*, the date of acceptance and the time of the draft determine the day of maturity. Thus, if a draft is dated May 16, accepted May 20, and payable sixty days after sight, it would mature or be due 63 (including 3 days of grace) days after May 20, or July 22. If payable 60 days after *date*, it would mature 63 days after May 16, or July 18. It is not necessary to present for acceptance drafts drawn a certain time after date, but as a courtesy to the drawee, it is usually done.

9. Days of grace are allowed on drafts according to the custom of the place where they are payable. The statute of New York forbids grace on all sight drafts, no matter on whom drawn, and on all time drafts which appear on their face to be drawn "upon any bank, or upon any banking association or individual banker, carrying on the banking business under the act to authorize the business of banking."

EXAMPLES.

354. *1.* How much would be due on Note 1, Art. **352**, Jan. 1, 1882, finding the time by compound subtraction?

2. How much would be due on Note 2, Art. **352**, at its maturity? How much March 1, 1883? Supposing the rate of interest was omitted in the note, how much would be due May 4, 1882?

3. Ninety days after June 21 is what date?

OPERATIONS.

		Or		
90		9		June.
9	June.	31		July.
81		31		Aug.
31	July.	71		
50		90		
31	Aug.	19		Sept.
19	Sept.			

ANALYSIS.—Subtract from the given number of days, the number of days remaining in June, and from this remainder, subtract successively the number of days in the following months until the remainder is equal to or less than the number of days in the next following month. The last remainder represents the required date.

Or, write the remaining number of days in June, and the number of days in a sufficient number of months to produce about the given number of days. Take their sum and subtract it (if possible) from the given number of days. The remainder will be the day of the following month representing the required date. If the sum is greater than the given number, subtract the

excess from the number of days in the last month written. The remainder will be the required date.

If the time be 30, 60, or 90 days, regard each 30 days as a calendar month, and correct by subtracting 1 day for each intervening month containing 31 days, and adding 2 days for February (in leap year 1 day). Thus 3 months after June 21 is Sept. 21, and by subtracting 2 days for July and August, the correct result is Sept. 19.

4. Supposing Note 3, Art. **352**, was payable 90 days from date, what would be its due date? The note as given not being paid at maturity, how much would be due Feb. 25, 1882, protest fees $2.10?

5. How much would settle Note 4, Art. **352**, Dec. 30, 1882?

6. If Draft 6, Art. **352**, was accepted June 19, 1881, what was the date of maturity?

BANK DISCOUNT.

355. Bank Discount is simple interest of a note, paid in advance, for the number of days the note has to run. It may be computed by any of the methods given for simple interest.

On notes without interest (the usual case of notes discounted at banks), bank discount is reckoned on their face, the amount due at maturity; on notes with interest, it is reckoned on the amount due at maturity, or their face plus the interest for the full time of the note.

356. The **Proceeds** of a note is the amount received by the holder from the bank when the note is discounted. It is the amount on which the discount is reckoned less the discount.

357. Call Loans.—Banks in the City of New York loan large amounts of money upon stocks, bonds, etc., as collateral security, payable on demand or on giving one day's notice. Such loans are called "call" or demand loans, and interest on them is paid at the end of the time.

358. The time to be reckoned on a loan or note is exclusive of the day of date, but includes the day of maturity or payment. Thus, in discounting a note in the City of New York, Apr. 4, which would mature Apr. 24, the discount would be calculated for 20 days.

In Philadelphia, Baltimore and other cities it is the custom of banks in finding time to include both the day of discount and the day of maturity. Thus, the discount on the above note would be reckoned for 21 days.

359. Banks of the City of New York reckon discount both on the basis of 360 and 365 days to the year.

NOTE.—In April 1880, the author made a personal investigation of this subject among the 70 banks of the City of New York, and found that their methods were not uniform; some banks reckoning discount on the basis of 365 days to the year, and others on the basis of 360 days. It is the custom of brokers and dealers in commercial paper to reckon interest and discount on the basis of 360 days to the year. Below are given extracts from letters received from some of the above banks.

"In discounting notes, we reckon interest on the basis of 365 days to the year when at 6%; 360 when at a rate lower than legal interest."

"In buying paper from a broker, we reckon on the basis of 360 days, no matter what the rate of discount."

"If we buy notes absolutely—without any recourse to the seller—as we frequently do of note-brokers and dealers in commercial paper—the usage is for banks to take, and brokers to allow, interest for the days to run to maturity on the basis of 360 days to the year."

"In discounting notes we reckon interest on the basis of 365 days to the year, while in making 'Call Loans' (**357**) the basis of reckoning is 360 days."

"All business with 'Wall St.' on stock loans, whether on demand or time, is calculated on the basis of 360 days to the year."

EXAMPLES.

360. Find the date of maturity and proceeds of the following notes:

(*1.*)

$10000. NEW YORK, *July 16, 1881.*

Four months after date, I promise to pay to the order of FISK & HATCH, Ten Thousand Dollars, at the First National Bank, value received.

S. D. BABCOCK.

Discounted July 16, 1881, at 6%.

ANALYSIS.—The note is due 4 months (**353**, 6) and 3 days (days of grace, **353**) after July 16, or Nov. 19. From the day of discount (July 16) to the day of maturity (Nov. 19) there are 126 days.

The interest of $10000 for 126 days at 6%, if reckoned on the basis of 360 days to the year, is $210, and the proceeds are $10000 less $210, or $9790.

The interest on the basis of 365 days to the year would be $2.88 less, or $207.12, and the proceeds would be $9792.88.

If the note was discounted Sept. 1, the interest or discount would be reckoned for 79 days (Sept. 1 to Nov. 19).

(*2.*)

$8000. BROOKLYN, N. Y., *July 16, 1881.*

Ninety days from date, I promise to pay S. B. CHITTENDEN, or order, Eight Thousand Dollars, value received.

A. A. LOW.

Discounted Aug. 31, 1881, at 6%.

ANALYSIS.—The note is due 93 days (**353**, 6) after July 16, or Oct. 17. Compute the discount for 47 days (Aug. 31 to Oct. 17) on $8000.

If the note had been discounted July 16, the date of the note, the interest would have been computed for 93 days, the full time of the note.

NOTE.—The results of the following examples will be given on the basis of both 360 and 365 days to the year.

No.	Date of Note.	Time.	Face.	Date of Discount.	Rate of Discount
3	Jan. 24.	90 days	$1200	Jan. 24.....	6%
4	May 18.....	3 mo.	$5280	May 18.....	6%
5	Aug. 31.....	60 days	$2560	Aug. 31.....	8%
6	June 4.....	4 mo.	$3756	June 4.....	7%
7	Oct. 16.	30 days	$6425	Oct. 16.....	5%
8	Mar. 13.....	6 mo.	$8375	Mar. 13.....	5½%
9	May 29.....	3 mo.	$4500	July 7.....	10%
10	July 27.....	60 days	$8240	Sept. 2.....	6%
11	Mar. 28.....	90 days	$4324	Apr. 14.....	5½%
12	May 27	6 mo.	$4885	Aug. 15.....	8%
13	Jan. 3.....	120 days	$9000	Feb. 28.....	6%
14	Sept. 12.	4 mo.	$5600	Oct. 14.	7%
15	Nov. 1.....	90 days	$6000	Nov. 28.....	5⅛%

Required the proceeds and date of maturity of the following notes discounted (360 days to the year) through a broker, his commission being ¼% of the face of the notes.

No.	Date of Note.	Time.	Face.	Date of Discount.	Rate of Discount
16	Feb. 21..... ..	4 mo.	$10000	Feb. 21.....	4⅛%
17	June 8.....	4 mo.	$6000	June 12.....	4⅛%
18	Jan. 10.....	4 mo.	$6000	Jan. 10.....	4¼%
19	Mar. 3.....	6 mo.	$8775	Apr. 30.....	4¾%

20. What were the proceeds of Note 3, Art. **352**, if discounted Dec. 16, 1881, at the legal rate?

21. Find the date of maturity and proceeds of a note of $5000, payable 60 days from date, dated and discounted at a Philadelphia bank, Aug. 3. (See Art. **358**.)

22. Find the date of maturity and proceeds of a note of $3750, payable 60 days from date, dated and discounted at a Maryland bank, Jan. 31, 1882.

23. A broker discounts a note payable in 4 months at 4¾%, and charges ¼% brokerage. This is equivalent to what rate of interest per annum, making no allowance for the days of grace?

24. A merchant can discount a note at his bank at 6%, 365 days to the year, or through a broker at 4¾%, 360 days to the year, broker's commission ¼%. How much better is the latter method on a note of $10000, payable in 4 months, dated and discounted May 21?

Find the date of maturity and proceeds of the following interest-bearing notes:

(*25.*)

$3000. ALBANY, N. Y., *September 16, 1881.*

Four months after date, I promise to pay W. J. KLINE or order, Three Thousand Dollars, with interest at 5%, value received.

J. M. THOMAS.

Discounted Nov. 3, 1881, at 6%.

NOTE.—Compute the discount at 6% for 77 days (Nov. 3 to Jan. 19) on the amount due at maturity ($3000 plus the interest of $3000 for 4 months and 3 days at 5%).

26. A note dated May 27, 1879, payable in 3 months, for $3750, with interest at 7%; discounted May 27, 1879, at 8%.

27. A note dated Jan. 16, 1879, payable in 4 months, for $1632, with interest at 6%; discounted Mar. 5, 1879, at 7%.

28. A note dated Oct. 12, 1878, payable in 6 months, for $875, with interest at 7%; discounted Jan. 10, 1879, at 10%.

29. For what amount must a note be given for 60 days to afford $1000 proceeds, if discounted at 6%?

ANALYSIS.—The proceeds of any note is as many times greater than the proceeds of $1, as the face of the note is greater than $1. If a note of $1 is discounted for 63 days, at 6%, it will afford $.9895 proceeds; to afford $1000 proceeds, the face of the note must be as many times $1, as $.9895 is contained times in $1000, or $1010.61.

The following approximate method is generally used by business men: *To the given proceeds, add the interest for the given time.*

The interest of $1000 for 63 days is $10.50. $1000 + $10.50 = $1010.50. Since the interest is reckoned on the proceeds instead of the face of the note, the error, 11 cents, is equivalent to the interest of the interest ($10.50) for the given time.

Where greater accuracy is required, the necessary correction may be made. The interest of $10.50 for 63 days is 11 cents. $1010.50 + $.11 = $1010.61.

30. A owes B $1500; how large a 90-day note must A give B that when discounted at a bank at 6%, the proceeds will be sufficient to pay the debt?

31. I hold a note of $3000 against Mr. C., which he pays by giving a new note at 90 days for $1500, and the balance, including the discount on the new note, in cash. Required the amount of cash paid.

32. A merchant having $8000 to pay, gets a note for $5000, that will mature in 40 days, discounted at a bank at 6%. How large a note must he draw, payable in 90 days, for discount at the same rate, that the proceeds of the two notes may enable him to meet his payment?

PARTIAL PAYMENTS.

361. Partial Payments are payments in part of a note, mortgage, or other debt, made at different times.

362. Indorsements are the acknowledgments of the payments, written on the back of the note, mortgage, etc., and stating the amount and date of the payment.

Special receipts are sometimes given for such payments.

UNITED STATES RULE.

363. Ex. How much would be due Sept. 1, 1882, on a note of $600, dated March 1, 1882, with interest at 6%? Suppose a payment of $100 be made Sept. 1, 1882, to pay the interest and part of the principal, how much would then be due? *Ans.* $518.

Ex. How much would be required to settle the above note Jan. 1, 1883, the balance of $518 remaining on interest at the same rate from Sept. 1, 1882? *Ans.* $528.36.

Ex. Find the amount due on the following note, Jan. 19, 1885:

$1000. BOSTON, MASS., *Aug. 1, 1881.*

One year after date, I promise to pay JORDAN, MARSH & Co., or order, One Thousand Dollars, for value received, with interest from date, at 6 per cent.

ALEXANDER H. RICE.

On this note are the following indorsements:

Received Apr. 21, 1882, $200. Received Aug. 1, 1883, $100.
Received Dec. 1, 1882, $25. Received July 7, 1884, $400.

NOTE.—The method given in the following operation, is that adopted by the Supreme Court of the United States, and has been made the legal method of nearly all the States. By the United States Rule, as this is generally called, settlements are made whenever the payments are equal to or exceed the interest due; if the payment exceeds the interest, it is applied first to discharge the interest, and the surplus is applied towards paying the principal; if the payment is less than the interest, it is not applied until the payments, taken together, are sufficient to pay all interest due; since no unpaid interest is added to the principal to draw interest, a new principal can never be greater than the preceding principal.

OPERATION.

Face of note, or principal, from Aug. 1, 1881		$1000
Interest from Aug. 1, 1881, to Apr. 21, 1882 (8 *mo.* 20 *da.*), added		43.33
Amount, Apr. 21, 1882,		1043.33
First payment, Apr. 21, 1882,		200.00
New principal from Apr. 21, 1882		843.33
Interest of $843.33 from Apr. 21, 1882, to Dec. 1, 1882, (7 *mo.* 10 *da.*)	$30.92	
(Interest exceeds the payment, and a new principal is not formed.)		
Interest of $843.33 from Dec. 1, 1882, to Aug. 1, 1883, (8 *mo.*)	33.73	64.65*
[Payments $125 ($25 + $100), now greater than the interest due ($64.65)].		
Amount, Aug. 1, 1883,		907.98
Second and third payments, $25 + $100		125
New principal from Aug. 1, 1883		782.98

* In many cases it can be determined mentally in advance whether the payment is greater or less than the interest. In this case the interest could be taken at once from Apr. 21, 1882, to Aug. 1, 1883 (1 *yr.* 3 *mo.* 10 *da.*), since it is evident that the payment ($25) is less than the interest of $843.33 for 7 *mo.* 10 *da.* (The interest of $800 for 7 *mo.* is $3\frac{1}{2} \times \$8$, or $28, and it would be more on $843.33 for 7 *mo.* 10 *da.*) If it is doubtful whether the payment is greater or less than the interest, perform all the work.

New principal from Aug. 1, 1883	$782.98
Interest of $782.98 from Aug. 1, 1883, to July 7, 1884 (11 *mo.* 6 *da.*)	43.85
Amount, July 7, 1884, ...	826.83
Fourth payment, July 7, 1884, ...	400
New principal from July 7, 1884	426.83
Interest of $426.83 from July 7, 1884, to Jan. 19, 1885 (6 *mo.* 12 *da.*)	13.66
Amount due Jan. 19, 1885, the final day of settlement, . *Ans.*	$440.49

364. UNITED STATES RULE.—*Find the amount of the given principal to the time when the payment or the sum of the payments exceeds the interest due; subtract from this amount the payment or the sum of the payments. Treat the remainder as a new principal, and proceed as before, to the time of settlement.*

EXAMPLES.

365. NOTES.—1. In the following examples, find the time by compound subtraction.

2. In the first five examples, all the payments exceed the interest.

$1680. TRENTON, N. J., *Oct. 9, 1880.*

1. On demand, I promise to pay COOPER, HEWITT & Co., or order, Sixteen Hundred Eighty Dollars. Value received.

JOHN A. ROEBLING.

On this note were indorsed the following payments:
Dec. 21, 1881, received $289.12. June 9, 1883, received $991.50.
How much was due Jan. 30, 1884?

2. On a note dated May 11, 1877, for $2000, are the following indorsements: — Aug. 6, 1879, $361; Feb. 11, 1880, $901.60; Nov. 2, 1882, $1000. What remained due Feb. 2, 1883, at 6%? At 5%?

3. On a note dated July 11, 1878, for $2400, are the following indorsements: — Sept. 17, 1879, $200; Jan. 29, 1880, $400; Nov. 29, 1881, $1150. What is the amount due Jan. 11, 1882, the interest being at 6%? At 7%?

4. On a mortgage for $1700, dated May 28, 1880, there was paid Nov. 12, 1880, $80; Sept. 20, 1881, $314; Jan. 2, 1882, $50; Apr. 17, 1882, $160. What was due Dec. 12, 1882, at 6%? At 8%?

5. On a note dated May 30, 1879, for $1666, are the following indorsements: — Apr. 9, 1880, $314; Nov. 4, 1880, $180; Aug. 25, 1881, $575. What was due June 30, 1882, at 6%? At 8%?

6. What was the amount due Oct. 17, 1881, upon a note for $1000, dated New York, Mar. 2, 1880, and on which the following payments were indorsed: — June 2, 1880, $80; Dec. 15, 1880, $20; May 2, 1881, $32; June 2, 1881, $60?

7. A note for $3600, dated May 12, 1880, bore the following indorsements: — Jan. 2, 1881, $255; Mar. 15, 1881, $225; June 3, 1881, $120; Aug. 6, 1881, $300; Feb. 3, 1882, $30. What was due June 2, 1882, at 6%? At 10%?

8. A note for $4000, dated Mar. 9, 1874, was indorsed as follows: — Jan. 18, 1876, $300; June 4, 1876, $400; Dec. 9, 1876, $1800; Sept. 1, 1879, $2000. How much had to be paid Jan. 1, 1880, to take up the note, at 6%? At 7%?

9. A mortgage of $6000 is dated May 9, 1877, on which there were the following payments: — July 15, 1878, $500; Nov. 27, 1878, $1000; June 1, 1879, $100; May 9, 1880, $275; Sept. 27, 1880, $2000. What was due Nov. 9, 1880, the interest being at 6%? At 12%?

10. What remained due June 3, 1882, on a note dated June 21, 1880, for $3300 with interest at the legal rate in Illinois, the following payments having been made? Oct. 9, 1880, $90; Jan. 15, 1881, $60; Mar. 27, 1881, $100; Aug. 6, 1881, $60; Dec. 15, 1881, $500. What remained due at the legal rate in Nevada?

Mercantile Rules.

366. The following methods are frequently used by merchants in finding the balance due on a note where partial payments have been made. They are similar to the methods in general use for finding the balance due on an open account ( ).

367. When the note runs for one year only, or less.

368. Rule.—*Compute the interest on the principal from the time it commenced to draw interest, and on each payment from the time it was made until the time of settlement, and deduct the amount of all the payments, including interest, from the amount of the principal and interest.*

NOTES.—1. This rule is used by some merchants when the note runs more than one year, although it is greatly to the disadvantage of the creditor, or holder of the note.

2. In solving examples by this rule, the different methods for finding time and interest, given in Art. **299**, are used. The results of the following examples will be given for the first method (Compound Subtraction and 360 days to the year).

EXAMPLES.

369. *1.* According to the mercantile rule, find the balance due May 12, 1882, on a note for $2400, dated July 12, 1881, on which the following payments have been made: Dec. 16, 1881, $40; Jan. 2, 1882, $100; Mar. 15, 1882, $150.

OPERATION.

Face of note, or principal, July 12, 1881,		$2400.00
Interest on the same to May 12, 1882 (10 *mo.*)		120.00
Amount, May 12, 1882		2520.00
First payment, Dec. 16, 1881,	$40.00	
Interest on the same to May 12, 1882 (4 *mo.* 26 *da.*)	.97	
Second payment, Jan. 2, 1882,	100.00	
Interest on the same to May 12, 1882 (4 *mo.* 10 *da.*)	2.17	
Third payment, Mar. 15, 1882,	150.00	
Interest on the same to May 12, 1882 (1 *mo.* 27 *da.*)	1.42	294.56
Balance due May 12, 1882		$2225.44

2. On a note dated Jan. 13, 1882, for $1234, are the following indorsements: May 17, 1882, $234; June 16, 1882, $345; July 27, 1882, $123; Sept. 19, 1882, $135. What remained due Nov. 13, 1882, at 6%? At 7%?

3. A note for $1567, dated Jan. 14, 1881, bore the following indorsements: Mar. 11, 1881, $50; May 13, 1881, $245; June 19, 1881, $374; Aug. 30, 1881, $412; Sept. 28, 1881, $316.40. What was due Jan. 1, 1882, at 6%? At 5%?

4. On a note dated Aug. 17, 1881, for $3300, were the following indorsements: — Dec. 18, 1881, $320; Feb. 5, 1882, $425; Apr. 13, 1882, $550; June 29, 1882, $630; July 16, 1882, $375; Aug. 1, 1882, $500. What amount was due Aug. 17, 1882, at 6%? At 10%?

5. On a note dated Mar. 16, 1883, for $2468, are the following indorsements: July 11, 1883, $750; Aug. 4, 1883, $428; Sept. 21, 1883, $150; Nov. 12, 1883, $170; Dec. 18, 1883, $128; Jan. 16, 1884, $224; Feb. 13, 1884, $600. What is the amount due Mar. 6, 1884, at 6%? At 8%?

370. When the note runs for more than one year.

371. Since it is the custom of merchants and bankers to balance their accounts annually, the following method is used by them in computing the balance due on a note when it runs more than one year.

It is equivalent to finding the balance due yearly by the previous rule, and treating the balance as a new principal. The periodical settlements are made annually, semi-annually, or quarterly, depending upon the custom of the merchant or banker in balancing his accounts. Some merchants make the end of the business year, Jan. 1 or July 1, the periodical rest, or date of settlement for notes and accounts.

When payments are made yearly greater than the interest due, this rule is the same as the New Hampshire rule for notes "with interest annually."

372. RULE.—*Find the amount of the principal for one year; also of each payment made during the year from the time the payment was made to the end of the year (1 yr. from the date of the note). From the amount of the principal, subtract the sum of the payments, including interest. With the remainder as a new principal, proceed thus for each entire year that follows, and for the interval between the end of the last year and the final date of settlement.*

EXAMPLES.

373. *1.* By the above rule, find the balance due Jan. 19, 1885, at 6%, on a note for $2400 dated Aug. 1, 1881, on which the following payments have been made: — Apr. 21, 1882, $200; Dec. 1, 1882, $25; Aug. 1, 1883, $100; July 7, 1884, $400. (Time by Compound Subtraction.)

OPERATION.

Face of note, or principal, Aug. 1, 1881,		$2400.00
Interest on the same for 1 year,		144.00
Amount, Aug. 1, 1882,		2544.00
First payment, Apr. 21, 1882,	$200.00	
Interest on the same to Aug. 1, 1882 (3 *mo.* 10 *da.*)	3.33	203.33
Balance and new principal, Aug. 1, 1882,		2340.67
Interest on the same for 1 year,		140.44
Amount, Aug. 1, 1883,		2481.11

Amount, Aug. 1, 1883,		2481.11
Second payment, Dec. 1, 1882,	$25.00	
Interest on the same to Aug. 1, 1883 (8 *mo.*)	1.00	
Third payment, Aug. 1, 1883,	100.00	126.00
Balance and new principal, Aug. 1, 1883,		2355.11
Interest on the same for 1 year,		141.31
Amount, Aug. 1, 1884,		2496.42
Fourth payment, July 7, 1884,	$400.00	
Interest on the same to Aug. 1, 1884 (24 *da.*)	1.60	401.60
Balance and new principal, Aug. 1, 1884,		2094.82
Interest on the same to date of settlement, Jan. 19, 1885 (5 *mo.* 18 *da.*)		58.65
Balance due Jan. 19, 1885,		$2153.47

2–10. Solve Examples 2–10, Art. **365**, according to the mercantile rule.

CONNECTICUT RULE.

374. The following rule for computing interest on obligations, where one or more payments have been made, was established by the Superior Court of Connecticut, March, 1784. (Kirby's Reports, page 49.)

375. RULE.—*I. Compute the interest to the time of the first payment; if that be one year, or more, from the time the interest commenced, add it to the principal, and deduct the payment from the sum total. If there be after-payments made, compute the interest on the balance due to the next payment, and then deduct the payment as above; and in like manner from one payment to another, till all the payments are absorbed; provided the time between one payment and another be one year or more.*

II. But if any payment be made before one year's interest hath accrued, then compute the interest on the principal sum due on the obligation for one year, add it to the principal, and compute the interest on the sum paid, from the time it was paid, up to the end of the year; add it to the sum paid, and deduct that sum from the principal and interest added as above.

III. If any payment be made of a less sum than the interest arisen at the time of such payment, no interest

is to be computed, but only on the principal sum for any period.

NOTES.—1. Should the final date of settlement be less than one year from the last date of settlement, compute the interest on the principal and the payments, if any, to the final date of settlement.

2. When the time between the payments is one year or more, and the payments exceed the interest due, the Connecticut Rule is the same as the U. S. Rule (**364**). When the time between the payments is less than one year, and the payment exceeds the interest due at its date, the settlement is made by the first Mercantile Rule (**368**).

EXAMPLES.

376. *1.* According to the law of Connecticut, how much is due June 1, 1885, on a note dated Aug. 1, 1881, for $1000, the following payments having been made? Apr. 21, 1882, $100; Dec. 1, 1883, $300; July 1, 1884, $20; Sept. 1, 1884, $200; Mar. 1, 1885, $300.

OPERATION.

Face of note, or principal,		$1000.00
Interest on the same for 1 year,		60.00
Amount, Aug. 1, 1882,		1060.00
First payment, Apr. 21, 1882,	$100.00	
Interest on the same to Aug. 1, 1882 (3 *mo.* 10 *da.*)	1.67	101.67
Balance and new principal, Aug. 1, 1882,		958.33
Interest to date of next payment, Dec. 1, 1883 (1 *yr.* 4 *mo.*)		76.67
Amount, Dec. 1, 1883,		1035.00
Second payment, Dec. 1, 1883,		300.00
Balance and new principal, Dec. 1, 1883,		735.00
Interest on the same for 1 year,		44.10
Amount, Dec. 1, 1884,		779.10
Third payment, July 1, 1884 (less than interest due)	$20.00	
Fourth payment, Sept. 1, 1884,	200.00	
Interest on the same to Dec. 1, 1884 (3 *mo.*)	3.00	223.00
Balance and new principal, Dec. 1, 1884,		556.10
Interest to final date of settlement, June 1, 1885 (6 *mo.*)		16.68
Amount June 1, 1885,		572.78
Fifth payment, Mar. 1, 1885,	$300.00	
Interest on same to June 1, 1885 (3 *mo.*)	4.50	304.50
Balance due at date of settlement, June 1, 1885,		268.28

2–10. Solve Examples 2–10, Art. **365**, according to the Connecticut Rule, at the legal rate (**298**).

NEW HAMPSHIRE RULE.*

377. According to the laws of New Hampshire, when payments are made upon a note, or other contract, by virtue of which interest is payable *annually* (**336**), they should be applied in the following order to the payment of—

1. Any simple interest that may have accrued upon the annual interest.
2. The annual interest.
3. The principal.

378. RULE.—*Find the interest due upon the principal and the annual interest at the annual rest (the time when the annual interest becomes due from year to year) next after the first payment. To the payment or payments made before this rest, add interest from the dates when they were made to the date of the rest, unless there is no interest due upon the principal, excepting that which is accruing during the year in which the payment or payments were made, and the payments together are less than the interest thus accruing, in which last case no interest is to be added to the payments. Deduct the payment or payments, with or without interest, as aforesaid, from the amount of principal, annual interest, and simple interest upon the annual interest due at the time of said rest, if such payment or payments equal or exceed the annual and simple interest then due; if less than such annual and simple interest, but greater than the simple interest due upon the annual interest, deduct the same from the sum of the annual and simple interest, and upon the balance of such annual interest find simple interest to the time when the next payment or payments are applied; if less than the simple interest due upon the annual interest, deduct the same from such simple interest and add the balance without interest to the other interest due at the time when the next payment or payments are applied.*

Proceed in like manner to the time of the first annual rest following the next payment, and to the end of the time required.

* From Report of State Superintendent of Public Instruction (1877).

EXAMPLES.

379. *1.* According to the law of New Hampshire, how much is due Jan. 1, 1886, on a note dated Jan. 1, 1880, for $2000, with interest annually at 6%, the following payments having been made: July 1, 1882, $500; Oct. 1, 1883, $50.

OPERATION.

First annual interest due Jan. 1, 1881, $120 + 2 *yr.* simple interest thereon, $14.40		$134.40
Second annual interest due Jan. 1, 1882, $120 + 1 *yr.* simple interest thereon, $7.20		127.20
Third annual interest due Jan. 1, 1883,		120.00
Principal		2000.00
		$2381.60
First payment, July 1, 1882,	$500	
Interest thereon from July 1, 1882, to Jan. 1, 1883,	15	515.00
Balance of principal due Jan. 1, 1883,		1866.60
Fourth annual interest of $1866.60, due Jan. 1, 1884,		112.00
Second payment, Oct. 1, 1883 (being less than the interest accruing during the year, it does not draw interest)		50.00
Balance of fourth annual interest unpaid		62.00
Fifth annual interest of $1866.60, due Jan. 1, 1885,		112
Sixth annual interest of $1866.60, due Jan. 1, 1886,		112
Simple interest on unpaid balance of fourth annual int. for 2 *yr.*		7.44
Simple interest on fifth annual interest for 1 year		6.72
Balance of principal		1866.60
Amount due Jan. 1, 1886		2166.76

2–10. Solve Examples 2–10, Art. **365**, according to the New Hampshire Rule, at the legal rate (**298**), supposing each note to contain the words "with interest annually."

VERMONT RULE.

380. The Vermont Rule for notes with interest is essentially the same as the United States Rule (**364**); and for notes "with interest annually," it is the same as the New Hampshire Rule, except that when payments are made on account of interest accruing but not yet due, they draw interest from the date they were made to the annual rest, whether they are greater or not than the interest accruing during the year.

Thus, by the Vermont Rule, the payment of $50, in the above example, would draw interest from Oct. 1, 1883 to Jan. 1, 1884, or 3 months. The unpaid balance of fourth annual interest would be $61.25 ($112 − $50.75).

RATIO AND PROPORTION.

DEFINITIONS.

381. **Ratio** is the relation of two numbers as expressed by the quotient of the first divided by the second. Thus the ratio of 6 to 3 is $6 \div 3$, or 2.

1. There is no ratio between quantities of different kinds; as 6 *bu.* and 3 *ft.* But a ratio exists between quantities of the same kind though of different denominations; as 6 *ft.* and 8 *in.* To express the ratio in such cases, the quantities must first be reduced to the same denomination. Thus, the ratio of 6 *ft.* to 8 *in.* is 72 *in.* $\div$ 8 *in.*, or 9.

2. The ratio between two numbers is denoted by placing a colon (the sign of division without the horizontal line) between them. Thus, the ratio of 6 to 3 is expressed 6 : 3.

382. The numbers whose ratio is expressed are the **terms** of the ratio. The two terms of a ratio form a **couplet**, the first of which is the **antecedent**, and the second, the **consequent**.

383. **Proportion** is an equality of ratios.

The ratio of 6 *yd.* to 3 *yd.* is 2, and the ratio of \$24 to \$12 is 2; hence from the two equal ratios the following proportion can be formed—6 *yd.* : 3 *yd.* = \$24 : \$12. This expression is read, "The ratio of 6 *yd.* to 3 *yd.* equals the ratio of \$24 to \$12." In place of the sign of equality (=), four dots (: :) are generally used; thus, 6 *yd.* : 3 *yd.* : : \$24 : \$12. The expression is also read, "6 *yd.* is to 3 *yd.* as \$24 is to \$12."

384. The first and fourth terms of a proportion are called the **extremes**; and the second and third, the **means**.

385. PRINCIPLES.—1. *The product of the means is equal to the product of the extremes.*

2. *A missing mean may be found by dividing the product of the extremes by the given mean.*

3. *A missing extreme may be found by dividing the product of the means by the given extreme.*

386. To solve examples by proportion.

Ex. If 24 hats cost $27, what will 32 hats cost?

ANALYSIS.—For convenience, make the fourth term the missing term, or the required answer. Since the third and fourth terms must be of the same denomination and the denomination of the answer will be dollars, take $27 as the third term. From the nature of the example, the answer will be more than $27, the third term, therefore make 32 hats the second term, and 24 hats the first term. The proportion will then be stated as follows: 24 hats : 32 hats :: $27 : x (Let x represent the unknown term). Multiplying 32 by 27, and dividing the product by 24, the fourth or missing term will be $36.

387. RULE.—*For convenience, take for the third term the number that may form a ratio with, or is of the same denomination as, the third term. If from the nature of the example, the answer is to be greater than the third term, make the greater of the two remaining terms (which must be of the same denomination) the second term; when not, make the smaller the second term. Then multiply the means (the second and third) together, and divide their product by the given extreme (the first term).*

NOTE.—After the example is stated, any factor of the given extreme may be cancelled with an equal factor of either of the means.

EXAMPLES.

388. Find the missing term (represented by x) in each of the following proportions (See Principles, Art. **385**):

1. $16 : x :: 24 : 18$.
2. $x : 27 :: 18 : 54$.
3. $32 : 27 :: x : 135$.
4. 24 *bu.* : 32 *bu.* :: $27 : x.
5. $48 : $75 :: $32 : x.
6. $375 : $144 :: 625 *lb.* : x.
7. $1728 : $288 :: $666 : x.
8. 144 *yd.* : 175 *yd.* :: $18 : x.
9. If 19 *yd.* of silk cost $28.50, what will 37 *yd.* cost?
10. If 64 *yd.* of carpet 36 *in.* wide will cover a floor, how many yards 27 *in.* wide will be required to cover the same floor?
11. A cane 3 *ft.* 3 *in.* high casts a shadow $5\frac{1}{2}$ *ft.* long; how long a shadow is cast by the steeple of a church which is 234 feet high?

12. If the freight of a long ton (**172,** 3) is 70 shillings, what is the freight of 16375 pounds?

13. The net assets of a bankrupt are $27675, and the liabilities $138375. How much must be paid to Mr. A, whom he owes $4800?

14. A building is insured in several companies for $28000. During a fire the building is damaged to the amount of $13500. What is the loss of company A, whose risk is $5000?

15. A invests in business $8450, and B $7200, and the gain or loss is divided according to the investments. What is each partner's share of gain, the total gain being $3474.30?

16. The U. S. gold dollar (**181, 183**) contains 23.22 ($25.8 - \frac{1}{10}$) grains of pure gold, and the standard silver dollar 371.25 ($412.5 - \frac{1}{10}$) grains of pure silver. What is the relative value of pure gold to pure silver?

17. The assessed value of the property of a certain town is $325000, and the total tax is $10238. How much is the tax of Mr. A, whose property is valued at $5700?

18. A bankrupt whose assets were $43225, pays 44 cents on a dollar; what did his debts amount to?

19. A cask holds 45 English (**167**) gallons; how many American gallons will it hold?

20. A company with a capital of $250000 divides $8750 among its stockholders. How much will be received by a stockholder who owns 36 100-dollar shares?

21. If a long ton of coal is worth $4.25, what is the value of a short ton?

22. If a farm valued at $4500 is taxed $26.24, what should be the tax on property valued at $23500?

23. If a man can walk a mile in 10 minutes, in what time can he walk a kilometer?

24. A piece of land 40 rods long and 4 rods wide contains an acre; what is the breadth of a piece 32 rods long, that is equivalent to an acre?

25. A merchant gains $625 by selling $12000 worth of goods; what amount must he sell to gain $8000?

26. Find the value of 6 *T.* (2240 *lb.*) 7 *cwt.* 2 *qr.* 20 *lb.* of iron at 85*s.* per ton.

27. How many feet of boards will be required for a fence 764 feet long, if 888 feet of boards are required for 288 feet?

INSURANCE.

DEFINITIONS.

389. Insurance is a contract by which one party (The Insurer or Underwriter) engages for a stipulated consideration (The Premium) to make up a loss which another may sustain.

Insurance is effected on property against loss or damage by fire and water, and on lives of persons. (For Life Insurance, see Art. .)

Insurance is also effected against accidents to persons, the breakage of plate-glass, the loss of live stock, and the fidelity of employees.

390. An **Insurance Company** is a company or corporation which insures against loss or damage.

Insurance companies usually make a specialty of a certain kind of insurance, as Fire, Marine, Life, Accident, etc. Certain companies combine Fire and Marine Insurance, while some of the large English companies have Fire, Marine, and Life departments.

391. Insurance companies may be classified according to principles of organization as follows: — 1, Stock; 2, Mutual; 3, Mixed, or Stock and Mutual.

Of the 188 Fire (126), Fire-Marine (49), and Marine (13) insurance companies doing business in the State of New York in 1879, 165 were Stock, 11 Mixed (Stock and Mutual), and 12 purely Mutual. Their net assets, Dec. 31, 1879, were $150,600,689; amount of risks in force, $6,997,419,444.

The above does not include many town and county co-operative insurance companies.

392. A **Stock Insurance Company** is one in which the capital is owned by individuals, called stockholders. They alone share the profits and are liable for the losses.

The business of a stock company and also of a mixed company, is managed by directors chosen by the stockholders. No policyholder, unless a stockholder, has any voice in any way in the election of the officers, or in the management of its business.

393. A **Mutual Insurance Company** is one in which there are no stockholders, and the profits and losses are shared among those who are insured (the policyholders).

Non-participating policies, the holders of which do not share in the profits or losses, are issued by certain mutual and mixed companies.

394. A **Mixed Insurance Company** is one which is conducted upon a combination of the stock and mutual plan.

Usually in a mixed company, all profits above a limited dividend to the stockholders are divided among the participating policyholders.

395. The **Policy** is the written contract between the Insurance Company (the Insurer or Underwriter) and the Insured. It contains a description of the property insured, the amount of the insurance, and the conditions under which the policy is issued, etc.

396. The **Premium** is the amount paid for the insurance.

1. Premium rates are expressed by giving the cost in cents of $100 insurance. The rate is sometimes expressed as a certain per cent. of the amount of the risk. Thus, a rate of 75 cents per $100 is equivalent to ¾%.

2. The premium rates depend upon the nature of the risk, and the length of time for which the policy is issued.

3. A fee of $1, or $1.25, is sometimes charged for the policy in addition to the premium.

397. An **Insurance Agent** is a person who represents an insurance company or several companies, and acts for them in soliciting business, collecting premiums, adjusting losses, etc.

398. An **Insurance Broker** is a person who effects insurance, for negotiating which he receives a commission or brokerage from the company taking the risk.

Brokers are regarded as agents of the insured, and not of the insurance company.

399. The **Surplus** of an insurance company is the excess of the assets over the liabilities (including capital and unearned premium).

FIRE INSURANCE.

400. Fire Insurance refers to insurance against loss or damage by fire.

Fire policies are usually issued for periods of from 1 to 5 years. Certain companies issue policies for longer periods. Of the outstanding risks of the largest insurance company of New York, Dec. 31, 1879, about 50% were for 1 year or less, 2% for 2 years, 28% for 3 years, 4% for 4 years, and 16% for 5 years.

401. Adjustment of Losses.—In an ordinary fire insurance policy, a person who insures will be paid the extent of his loss up to the amount of his insurance; but in policies containing the "average clause," the payment is such proportion of the loss as the amount of the insurance bears to the total value of the property.

1. The following is the usual form of the "average clause" above referred to: "It is a condition of this insurance, that if the whole value of the above described property, contained in any or all of the above mentioned buildings and premises, shall exceed the whole amount of insurance thereon, then, in case of loss or damage by fire, this policy shall contribute to the payment of said loss or damage in the proportion only that the whole amount of insurance on said property shall bear to the whole value of said property, in all of said buildings, at the time said loss or damage may occur."

2. Under a policy containing the "average clause," a person who insures $5000 on property worth $10000, would receive only $2500 in case of an actual loss of $5000; $1500 in a loss of $3000; $4000 in a loss of $8000.

3. Insurance companies usually reserve the privilege of replacing or repairing the damaged premises.

402. A Floating Policy is one which covers property stored in several buildings or places. The name is applied more particularly to policies which cover goods whose location may be changed in process of manufacture or in the ordinary course of business. The "average clause" is a usual condition of policies of this class.

403. Short Rates are rates for a term less than a year.

If an insurance policy is terminated at the request of the policyholder, the company retains the customary "short rates" for the time the policy has been in force; if terminated at the option of the company, a ratable proportion of the premium is refunded for the unexpired term of the policy.

MARINE INSURANCE.

404. Marine Insurance refers to insurance of vessels and their cargoes against the dangers of navigation.

1. Inland and Transit Insurance refer to insurance of merchandise while being transported from place to place either by rail or water routes, or both.

2. Policies on cargoes are issued for a certain voyage, or from port to port, and on vessels for a specified time or for a certain voyage.

3. The particular average clause is the clause which exempts the insurance company from the payment of any partial loss or particular average, unless it exceeds a certain per cent. of the value of the property. The particular average clause is sometimes applied to the value of each parcel or series of parcels, according to invoice numbers.

4. Insurance Certificates, showing that certain property has been insured, and stating the amount of the insurance and the name of the party abroad who is authorized to make the settlement, are issued by marine companies. They are negotiable, and are usually sent to the consignee of the merchandise to make the loss payable at the port of destination, and to otherwise facilitate the adjustment of the insurance in case of loss.

405. Adjustment of Losses.—In marine insurance, in case of loss or damage, the insurance company contributes such proportion of the loss as the amount of the insurance bears to the total value of the property.

1. The adjustment of marine policies in case of loss is on the same principle as the adjustment of fire policies containing the "average clause" (**401,** 1).

2. In the adjustment of marine losses, the pound sterling is usually estimated at $4.95.

406. An **Open Policy** is one upon which additional insurances may be entered at different times. It covers merchandise which may be shipped on "Vessel or Vessels" from "Ports and Places" to "Ports and Places," for amounts "as endorsed" and at rates "as agreed."

1. The date of the shipment, name of vessel, ports of shipment and destination, the amount of the insurance, rate, premium, and a description of the property are entered on the policy or in a pass-book, which is regarded as part of the policy. (See Ex. 29, Art. **407**.)

2. Open policies with pass-books attached and insuring merchandise against loss or damage by fire, are issued by fire insurance companies.

3. Open policies, which cover all risks whether accepted and endorsed on the policy or not, are issued to merchants who are receiving merchandise from foreign countries, and who do not always have a definite knowledge of the time and mode of shipment. Such policies usually contain the following clause: "The company are to be entitled to premiums at their usual rates on all shipments reported or not. It is warranted by the assured to report every shipment on the day of receiving advice thereof, or as soon thereafter as practicable, when the rate of premium shall be fixed by the President or Vice-President of the Company."

The above policies cover the invoice cost and 10% additional until the amount of the risk is endorsed on the policy or pass-book.

4. Open policies are sometimes issued which cover only such risks as may be accepted and endorsed on the policy by the company.

EXAMPLES.

407. *1.* A building was insured for \$2500 in one company at $1\frac{1}{4}\%$, and for \$5000 in another company at 125 cents. What was the total premium paid?

2. A cargo of goods was insured for \$9000 at $\frac{1}{4}\%$. What was the cost of the insurance, \$1.25 being charged for the policy?

3. What is the total premium of the following insurances: \$5000 at $1\frac{1}{2}\%$ for 2 years, \$7000 at 45¢ for 5 years, \$1500 at 1% for 4 years, \$2000 at 5% for 7 years, \$3500 at 45¢ for 1 year, \$2000 at 70¢ for 4 years, \$4000 at $1\frac{1}{4}\%$ for 5 years, \$2000 at 60¢ for 4 years, \$4500 at 25¢ for 2 years, \$3600 at 125¢ for 1 year, and \$3000 at 240¢ for 4 years?

4. \$20 were paid for an insurance of \$2500; what was the premium rate?

5. \$25.20 were paid for an insurance at the rate of 70¢ per \$100. What was the amount of the risk?

6. A factory was insured for \$7500 for 1 year at $2\frac{1}{2}\%$, stock for \$2500 at $2\frac{1}{2}\%$, and raw material for \$2500 at $1\frac{1}{4}\%$. What was the total premium?

7. What is the cost of insuring a house for \$5000 at the rate of 45¢ per \$100?

8. A cargo of merchandise was insured for \$6500 at $\frac{1}{8}\%$, including the risk of fire while on wharf awaiting shipment. What was the premium?

9. A building was insured Jan. 1, 1880, for \$2000, for 7 years, at 5%; what was the value of the unearned premium, Jan. 1, 1882?

10. A shipment of goods was insured in the Pacific Mutual Insurance Co. for \$9600 at 75¢ less 20% in lieu of scrip and interest. What was the net cost of the insurance?

11. A house was insured for \$5000 for 4 years at 60¢ per annum. The house was destroyed by fire. What was the actual loss of the company, making no allowance for interest?

12. Suppose the above house was worth \$8000. What was the actual loss of the owners?

13. A cargo of hides from Montevideo to New York having increased in value since the insurance was effected, the anticipated profits were insured for \$3000 at $1\frac{3}{4}\%$ less 20%. What was the premium?

14. A factory (worth \$3000) and its contents are insured for \$10000 as follows: \$2000 on building, \$3000 on machinery (worth \$5000), and \$5000 on stock (worth \$8000). The building is damaged by fire to the amount of \$1000, the machinery \$4000, and stock is a total loss. How much is the claim against the insurance company?

15. A cargo of goods valued at \$20000 was insured for \$12000. If the goods were damaged to the amount of \$15000, how much of the loss would be paid by the insurance company? (Art. **405.**)

16. A building is insured in several companies for \$60000, and is damaged by fire to the extent of \$24000. What per cent. of its risk is paid by each company?

17. A stock of goods was insured, May 1, for 1 year, for \$6000, at 90¢. The policy was cancelled Nov. 1, at the request of the insured. How much was the return premium, the short rate for 6 months being 63¢? How much would have been returned by the company, if the policy had been cancelled at its request?

18. A quantity of merchandise valued at \$6000 is insured for \$5000. It is damaged by fire to the amount of \$1728. How much of the loss is paid by the insurance company, the policy containing the "average clause" (**401**)?

19. What was paid for insuring a cargo of merchandise for \$8750 at $\frac{1}{8}\%$ less 20%?

20. A marine rate of $\frac{1}{2}\%$ for a voyage of 10 days is equivalent to what rate per annum?

21. What were the average net assets for every \$100 insured of the F. F.-M., and M. Ins. Cos., doing business in the State of New York in 1879? (See Art. **391**, Note.)

22. A factory and its contents are insured for $5000 in company M, $5000 in N, $5000 in O, $4000 in P, and $2500 in each of the following companies: Q, R, S, T, U, V, W, X, Y, and Z. What was the total premium, the rate being 2% less 10% ?

23. The above insurance covered the following property: $4000 on building marked A on plan, $4000 on B, $5000 on C, $500 on D, $500 on E, $3500 on stock and materials in building marked A on plan, $8000 on machinery, etc., in A, $11500 on stock and materials in B and C, $4000 on machinery, etc., in B and C, $2500 on horses in D, $500 on harness, hay, feed, etc., in D. Suppose building A and its contents were totally destroyed by fire, what would be the loss of company M? Of P? Of T?

NOTE.—The above insurance was divided pro rata among the several companies, each policy designating the exact amount on each building, etc.

24. In the above example, what is the amount of the risk of company M on the building marked A on plan? On C?

25. The net invoice value of a quantity of goods is $6325, and the insured value $6500. The insured value is what per cent. greater than the invoice value?

26. A quantity of merchandise valued at $9035, is insured for $9000. What is the insurance on part of the same, the estimated value being $2638?

27. If 500 packages of merchandise are insured for $2627.78, what is the insurance on 60 packages?

28. The estimated sound value of a quantity of merchandise, damaged at sea, was $328.55, and the proceeds when sold at auction, $299.35. How much of the loss was shared by the Insurance Co., the insurance having been $315.33?

29. Make the extensions of the following "open policy" and find the total amount.

Date.	Name of vessel.	From.	To.	On.	Amount insured.	Rate.	Amount of premium.
1881.							
Sept. 2	Othello.	N. Y. via Hull.	Stockholm.	50 Ba. Mdse.	5100	$1\frac{1}{8}$	**.**
" 7	Algeria.	New York.	Liverpool.	68 " "	6675	$\frac{1}{2}$	**.**
" 16	Germanic.	New York.	Liverpool.	92 " "	13500	$\frac{1}{2}$	**.**
" 17	Rialto.	N. Y. via Hull.	Christiania.	6 " "	600	1	*.
" 23	Otranto.	N. Y. via Hull.	Orebro.	30 " "	2700	$1\frac{1}{8}$	**.**
							$***.**
						Less 20%	**.**
							***.**

30. Claim of Shultz, Southwick & Co., for partial loss on merchandise, per "Lessing," from New York to Hamburg, Feb. 24, 1882.

Insured value of cargo $10000
Net invoice value 9696
Advance . . . . . *** = *.***%.

Marks and Numbers.	No. of pkgs. shipped.	Invoice weight	Invoice value.	No. of pkgs. damaged.	Proportional invoice weight.	Proportional invoice value.	Advance at *.***%.	Insured value of damaged.	Sound wt., Germ. lbs. †
H R									
251	550	9497	28½¢	233	4023	1146.55			3621
2	150	3357	28¼	46	1029	293.26			927
3	275	4702	27⅛	118	2018	554.95			1817
*	*	*	*	*	*	*			*
	2001			1071		$5137.03	$***.**	$****.**	16792

Sound weight 16792 *lbs.* — 223 *lbs.* (Tare) = ***** *lbs.* @ 1.35 Rm. = Rm. ****.**
Less discount 1% . . . . ***.**
Sound value, . . . Rm. *****.**
Gross proceeds at auction . 14729.81
Loss . . . . . Rm. ****.**

Loss = **.**% of sound value.
Insured value of damaged $****.** @ **.**% = loss $****.**
Charges, . . . Rm. 201.32
Inspection 185.44
Agents' fees, . . . 223 Rm. ***.** @ 24¢ = $***.**
Total claim . . . . . $****.**

31. The total paid-up capital of the joint-stock fire and fire-marine companies doing business in the State of New York (excepting foreign companies), Dec. 31, 1879, was $50,992,220, and the surplus $34,998,146. The total surplus was what per cent. of the total capital?

32. The above companies, with the exception of the New York Mutuals (6), during the year 1879, received $69,657,129 in gross premiums for insuring $7,991,450,000. What was the average premium for every $100 insured?

33. Dec. 31, 1879, the capital stock of the Insurance Co. of N. A., Philadelphia, Pa., was $2,000,000; surplus, $2,338,378; dividend paid during 1879, $400,000. The surplus is what per cent. of the capital stock? The dividend is what per cent. of the capital, and of the capital and surplus?

† See Art. 243.

EXCHANGE.

DEFINITIONS.

408. Exchange is the system by which merchants in distant places discharge their debts to each other without the transmission of money.

Suppose, for example, A of New York owes B of Chicago $1000 for grain, and C of Chicago owes D of New York $1000 for dry goods. The two debts may be discharged by means of one draft or bill of exchange without the transmission of money. Thus, B of Chicago draws on A of New York for $1000, and sells the draft to C of Chicago who remits it to D of New York. D of New York presents the draft to A of New York for acceptance or payment, and thus both debts are cancelled. There is in effect a setting-off or exchange of one debt for the other.

The business of exchange is usually conducted through the medium of banks and bankers, who buy commercial bills and transmit them for credit to the places on which they are drawn. They also sell their own drafts on their correspondents in any amounts demanded.

409. A **Bill of Exchange**, or **Draft**, is an order or request addressed by one person (the Drawer) to another (the Drawee), directing the payment of a specified sum of money to a third person (the Payee) or to his order. It is issued at one place and payable at another. (See Art. **352**, 5–6.)

For brevity, bills of exchange are frequently called "exchange."

According to the laws of most States, drafts drawn in one State and payable in another, are termed *foreign* bills of exchange. For the purposes of this book, the term "domestic exchange" will be applied to bills drawn and payable in the United States.

410. Bills of exchange are of two kinds, Inland or Domestic, and Foreign.

411. A **Domestic** or **Inland Bill of Exchange** is one which is payable in the same country in which it is drawn.

412. A **Foreign Bill of Exchange** is one which is payable in a different country from the one in which it is drawn; as a draft drawn in the United States and payable in England.

413. When drafts sell for more than their face value, exchange is above par or at a premium; when for less than their face, below par or at a discount.

When Chicago owes New York the same amount that New York owes Chicago, exchange will be at par; that is, drafts will sell at their face value. When Chicago owes New York more than New York owes Chicago, drafts on New York will sell at a premium; there will be more buyers of exchange than sellers, and drafts will sell for more than their face value. When Chicago owes New York less than New York owes Chicago, the demand in Chicago for drafts on New York will be less than the supply, and drafts will sell for less than their face value, or at a discount.

DOMESTIC EXCHANGE.

414. **Domestic** or **Inland Exchange** relates to drafts drawn at one place on another in the same country.

415. The domestic exchanges on New York at the places named were quoted as follows, May 7, 1881: Savannah, $\frac{1}{8}$ @ $\frac{3}{8}$ premium; Charleston, $\frac{1}{8}$ @ $\frac{1}{4}$ premium; New Orleans, \$1.50 @ \$2.50 premium; St. Louis, 25 cents premium; Chicago, 50 @ 75 cents premium; and Boston, 25 cents discount.

1. At Savannah and Charleston the rates per cent. of the premium or discount are given. Thus, when exchange is quoted at $\frac{1}{4}$ premium, a draft of \$100 may be purchased for \$100$\frac{1}{4}$ (\$100.25).

2. At New Orleans, St. Louis, Chicago, and Boston, the premium or discount per \$1000 is given. Thus, a draft of \$1000 at \$2.50 premium may be purchased for \$1002.50. \$2.50 per \$1000 premium is equivalent to $\frac{1}{4}\%$ premium.

3. The selling rates are about $\frac{1}{8}\%$ (\$1.25) higher than the buying rates, and bankers' exchange is usually higher than commercial.

4. The rate of domestic exchange is limited by the cost of shipping gold or currency by express, and the premium or discount will not exceed this cost. Thus, if a merchant in Chicago is charged a premium of \$10 for a draft of \$10000, and he can send the currency by express for \$7.50, it will be to his advantage to remit by the latter method.

The following appeared in a New York financial paper, May 8, 1881, the date of the above quotations: — "The domestic exchanges at the West are sufficiently high to permit of a movement of funds Eastward, but at the East,

New York funds are still at a discount and some shipments of gold and currency continue to be made to the Eastern cities."

5. The preceding quotations refer to sight exchange. Time drafts are discounted in the same manner as promissory notes. In certain cases bankers in discounting notes and drafts payable in distant places, charge interest for the time required for the return of the money when the note or draft is paid; and in the case of drafts drawn a certain number of days after sight, bankers sometimes charge interest for the time required for the acceptance of the drafts. Thus, if a draft was drawn in New York on St. Louis and payable 60 days after sight, it would require, in the ordinary course of the mails, 3 days for the acceptance of the draft. The draft would be paid in 63 days (including the days of grace), and 3 days would elapse before the money would be returned to New York. The banker would be justified in charging interest for 69 days, the interval between the day he advanced the money in New York, and the day it was returned to him again. If the draft was drawn on San Francisco, fully 19 days (8 days for the acceptance, 3 days of grace, and 8 days for the return of the money) would be added to the time of the draft. Between New York and San Francisco and other distant places, money is frequently transferred by telegraph.

EXAMPLES.

416. *1.* What is the value in Savannah of a draft on New York for \$8750 at $\frac{3}{8}\%$ premium?

2. Find the cost in New Orleans of a draft on New York for \$8375 at \$2.50 premium.

Find the value of the following drafts:

Face.	Exchange.	Face.	Exchange.
3. \$5000,	$\frac{1}{8}\%$ premium.	*8.* \$4287.75,	15¢ discount.
4. \$4375,	$\frac{3}{8}\%$ discount.	*9.* \$3416.33,	25¢ premium.
5. \$8417,	$\frac{1}{8}\%$ premium.	*10.* \$2825.49,	\$1.25 discount.
6. \$9873,	$\frac{1}{2}\%$ premium.	*11.* \$9873.62,	\$2.50 premium.
7. \$5284,	$\frac{1}{4}\%$ discount.	*12.* \$8412.75,	75¢ discount.

13. A of Chicago buys cattle for B of New York to the amount of \$9858.07. How large a draft should be drawn on B, so that when sold at a discount of 50¢ ($\frac{1}{2}\%$), the proceeds would be sufficient to pay the bill?

NOTE.—To find the face of a draft, instead of dividing the value of the draft by the rate of exchange (in the above example, .99$\frac{1}{2}$ or .9995), business men and bankers calculate the premium or discount on the value of the draft, and subtract or add it to the value as the case requires. Thus, in the above example, the discount would be $\frac{1}{2}$ of $\frac{1}{10}\%$ of \$9858.07, or \$4.93, which added to the given proceeds would produce the face \$9863. This method produces too small a result in all cases, the error being equivalent to the percentage of the premium or discount. In this example the error is less than $\frac{1}{4}$ cent.

For ordinary examples in business, the above method is sufficiently accurate. At $\frac{1}{2}\%$, or \$5.00 (a very high rate for domestic exchange) on a draft whose value is \$10000, the error would only be 25 cents. If greater accuracy is required, the necessary correction can be made by adding the percentage of the premium or discount. Thus, if the value of the draft is \$10000, and exchange is $\frac{1}{2}\%$ discount, the face would be \$10000 + \$50 ($\frac{1}{2}\%$ of \$10000) + \$0.25 ($\frac{1}{2}\%$ of \$50) = \$10050.25. If at $\frac{1}{2}\%$ premium, the face would be \$10000 − \$50 + \$0.25 = \$9950.25.

By the above method, find the face of the following drafts:

	Value.	Exchange.		Value.	Exchange.
14.	\$1876.16,	$\frac{1}{4}\%$ premium.	*19.*	\$7375,	25¢ premium.
15.	\$2437.75,	$\frac{1}{4}\%$ discount.	*20.*	\$9218,	50¢ discount.
16.	\$3342.38,	$\frac{1}{8}\%$ discount.	*21.*	\$6438,	\$1.00 premium.
17.	\$2238.42,	$\frac{1}{2}\%$ premium.	*22.*	\$9243,	\$1.25 premium.
18.	\$8175.50,	$\frac{3}{8}\%$ premium.	*23.*	\$5280.	75¢ discount.

24. A of New Orleans being indebted to B of New York \$9316.75, forwards to him a check on a New Orleans bank for that amount, to cash which B is obliged to allow a discount of $\frac{1}{4}\%$. How much does A still owe B, and for what amount should the check have been drawn to net B the amount due?

25. What is the value of a draft on New York for \$3000, payable in 60 days (63 days) after date (**353**, 8), exchange being $\frac{1}{8}\%$ premium, and interest 6%?

NOTE.—From the face of the draft, subtract the interest, and to the result add the exchange.

26. Find the proceeds of a draft drawn at Chicago on New York for \$12000, and payable 90 days after sight, exchange 50¢, interest 5%, and allowing 3 days additional for the acceptance of the draft.

27. A banker in New York discounts a draft for \$8000, payable in San Francisco 60 days after sight; what would be the proceeds, exchange being $\frac{1}{2}\%$ discount, interest 6%, and allowing 8 days for the acceptance and 8 days for the return of the money?

28. A merchant paid \$6920.64 in Charleston for a sight draft of \$6912; what was the rate of exchange?

29. A commission merchant sold 13475 pounds of leather at $26\frac{3}{4}$ cents a pound. If his commission is 5%, and exchange $\frac{1}{8}\%$, how large a draft can he buy to remit to the consignor?

30. How large a 60-days' draft must I draw, so that when sold it will produce \$10000, exchange $\frac{1}{8}\%$ discount, interest 6%?

FOREIGN EXCHANGE.

417. Foreign Exchange relates to drafts or bills of exchange drawn in one country and payable in another.

Foreign bills of exchange are usually drawn in the moneys of account of the countries in which they are payable. Thus, drafts on England are usually drawn in pounds, shillings, and pence; on France, Belgium and Switzerland, in francs; on Germany, in marks; on the Netherlands (Holland), in guilders.

Foreign bills of exchange are usually drawn at sight (3 days) or at sixty (63 days) days' sight. Sight drafts are frequently called "short" exchange, and 60 day drafts, "long" exchange. "Long" exchange is sold at a rate below that for "short" exchange, sufficient to equalize the difference in interest between the dates of maturity of the two classes of bills.

418. To secure safety and speed in the transmission of foreign bills of exchange, they are drawn in sets of two or three of the same tenor and date. The separate bills are sent by different steamers, and when any one of them is paid, the others become void. Some merchants send only the first and second, and preserve the third.

SET OF EXCHANGE.

(1.)

EXCHANGE FOR £1000. NEW YORK, *May 16, 1882.*

Sixty days after sight of this FIRST of Exchange (Second and Third unpaid), pay to the order of A. T. STEWART & Co., One Thousand Pounds Sterling, value received, and charge the same to account of

No. 1738. BROWN BROTHERS & Co.

To BROWN, SHIPLEY & Co.,
London, England.

(2.)

EXCHANGE FOR £1000. NEW YORK, *May 16, 1882.*

Sixty days after sight of this SECOND of Exchange (First and Third unpaid), pay to the order of A. T. STEWART & Co., One Thousand Pounds Sterling, value received, and charge the same to account of

No. 1738. BROWN BROTHERS & Co.

To BROWN, SHIPLEY & Co.,
London, England.

(3.)

EXCHANGE FOR £1000. NEW YORK, *May 16, 1882.*

Sixty days after sight of this THIRD of Exchange (First and Second unpaid), pay to the order of A. T. STEWART & Co., One Thousand Pounds Sterling, value received, and charge the same to account of

No. 1738. BROWN BROTHERS & Co.

To BROWN, SHIPLEY & Co.,
London, England.

419. A Letter of Credit is an instrument issued by a banker and addressed to bankers generally, by which the holder may draw funds at different places and in amounts to suit his convenience, the total amount drawn not exceeding the limit of the letter of credit.

A *bill of exchange* is payable at a certain place, at a certain fixed time, and for a certain amount, while a *letter of credit* is payable at different places, at different times, and in different amounts.

A person, who intends to travel in foreign countries, may procure a letter of credit by depositing either cash or securities with a foreign exchange banker for the amount of the letter. When the American banker is notified of the payment of the traveler's drafts in London, he debits the account of the holder of the letter of credit with the amount drawn and the charges, at the current rate of exchange. A small rate of interest is allowed on the account, and a settlement is made on the return of the traveler.

If a person has business connections, he may avoid making a deposit by having some commercial firm sign a bond as security. By this method, when the New York banker is notified of the payment of the traveler's draft in London, he immediately draws a sight draft (**420**) for the amount and the charges (**420**) on the traveler's representative, and no account is kept with the traveler on the books of the banker. In this case, a settlement is made with the commercial house on the return of the traveler.

The holder of a letter of credit desiring funds, presents it to a banker at the place he may be visiting. The banker will prepare a sight draft, which the holder of the letter will sign, on the London banker mentioned in the letter of credit. If the signature on the draft and on the letter of credit correspond, the draft will be cashed by the banker at the current rate of exchange. The bankers who cash the drafts of the holder of the letter, write the date of payment, their names, and the amounts drawn (in words and figures), on the back of the letter of credit. When the London banker pays the drafts, he immediately notifies the American banker (the issuer of the letter of credit). The foreign bankers mentioned as correspondents in a Letter of Credit are bound to honor the drafts of the holder; but other banks and agencies where the parties are known, are also free to respond.

BROWN BROTHERS & CO.'S CIRCULAR LETTER OF CREDIT.

No. $\frac{B}{B}$ 14506. NEW YORK, *June 2, 1881.*

GENTLEMEN:—We request that you will have the goodness to furnish MR. EUGENE HORTON, the bearer, whose signature is at foot, with any funds he may require to the extent of £1000 (say One Thousand Pounds Sterling), against his drafts upon MESSRS. BROWN, SHIPLEY & Co., London; each draft *must bear the number* (No. $\frac{B}{B}$ 14506) of this letter, and we engage that the same shall meet due honor.

Whatever sums MR. HORTON may take up, you will *please endorse on the back* of this Circular letter, which is to continue in force till June 2, 1882, from the present date, June 2, 1881.

We are respectfully, gentlemen,

Your obedient humble servants,
BROWN BROTHERS & Co.

The Signature of
EUGENE HORTON.

To MESSRS. THE BANKERS,
Mentioned on the third page of this Letter of Credit.

420. The following draft, drawn by the issuer of the letter of credit on the traveler's American representatives, shows the expense connected therewith:

		£.	s.	d.
No. 51931.	Draft dated Lucerne, July 20.	25		
	Commission @ 1%, ...		5	
Cr. $\frac{B}{B}$ 14506.	Interest for 33 days @ 5%, .		2	3
		25	7	3

NEW YORK, *Aug. 11, 1881.*

EXCHANGE FOR £25 7*s.* 3*d.*, at $\$4^{85}$ per £ = $\$122^{96}$.

On demand, pay this FIRST of Exchange (Second unpaid), to our order, the sum of Twenty-five Pounds $^{1}/_{3}$ Sterling, for value received by MR. EUGENE HORTON.

BROWN BROTHERS & Co.

To MESSRS. G. B. HORTON & Co.,
New York.

NOTES.—1. The commission is charged only on amounts drawn and not on the face of the letter of credit.

2. The interest charged is calculated to cover the time between the payment of the original draft in London and the maturity of a shortsight remittance from New York in reimbursement.

3. Exchange is charged at the current rate of sight exchange on London.

421. The **Intrinsic Par of Exchange** is the value of the monetary unit of one country expressed in that of another, and is based on the comparative fineness and weight of the coins, as determined by assay.

The intrinsic par of exchange between different countries and the United States, is given in Art. **192.**

422. The **Commercial Par of Exchange** is the market value in one country of the *coins* of another.

423. The **Commercial Rate of Exchange** is the market or buying and selling value in one country of the *drafts* on another.

1. In giving quotations of foreign exchange, no reference is made to the par value, the quotations being given by means of equivalents.

2. Premium or discount for exchange can not long exceed the transportation charges and insurance of shipping coin; for, if a merchant can ship gold cheaper than he can buy a bill of exchange, he will choose the former method of paying his indebtedness. When sight exchange is 4.84, gold can be imported at a small profit; and when sight exchange is 4.89¼, gold can be exported at a profit.

424. The quotations of foreign exchange, Apr. 20, 1881, were as follows:

Where payable.	60 days.	Sight.
London:		
Prime bankers'	4.81⅛	4.84
Good bankers' and prime commercial	4.81	4.83½
Documentary commercial	4.78½	4.81⅛
Cable transfers		4.84⅛
Paris (francs)	5.27⅛	5.24⅜
Antwerp (francs)	5.27⅛	5.24⅜
Swiss (francs)	5.26⅞	5.23⅛
Amsterdam (guilders)	.39⅝	.39⅞
Hamburg (reichsmarks)	.93⅜	.94⅜
Frankfort (reichsmarks)	.93⅜	.94⅜
Bremen (reichsmarks)	.93⅜	.94⅜
Berlin (reichsmarks)	.93⅜	.94⅜

In the preceding quotations, exchange is below par. (See intrinsic par values below, or in Art. **192**.) When exchange is above par, we are exporters of gold; when below par, we are importers of gold.

425. Exchange on England (Sterling exchange) is quoted by giving the value of £1 in dollars and cents.

Thus, when exchange is 4.84, a draft of £1 will cost $4.84; of £100, $484. The intrinsic par value of £1 is $4.8665 (**192**).

426. Exchange on France, Belgium, and Switzerland is quoted by giving the value of $1 in francs and centimes (hundredths of a franc).

Thus, when exchange is 5.27½, $1 will buy a bill of 5 francs and 27½ centimes; a draft of 1000 francs will cost $189.57 (1000 ÷ 5.27½). The intrinsic par value of 1 franc is $19\frac{3}{10}$ cents (**192**); of the equivalent exchange, 5.18⅞ (1.00 ÷ .193).

In French, Belgian, and Swiss exchange, the higher the apparent rate, the less the value of the draft. Thus, when exchange is 5.13, a draft of 1000 francs is worth $194.93, and each franc is worth $19\frac{49}{100}$ cents. When exchange is 5.26⅜, the same draft would be worth $189.98, and each franc 19 cents.

427. Exchange on Amsterdam (Netherlands) is quoted by giving the value of one guilder (gülden) or florin in U. S. cents.

The intrinsic par value of 1 guilder is $40\frac{2}{10}$ cents (**192**).

428. Exchange on Germany is quoted by giving the value of 4 reichsmarks in cents.

The intrinsic par value of 1 mark is $23\frac{8}{10}$ cents (**192**); of 4 marks $95\frac{2}{10}$ cents.

429. Documentary Exchange is a bill drawn by a shipper upon his consignee against merchandise shipped, accompanied by the bill of lading, "to order," and the insurance certificates, covering the property against which the bill is drawn.

430. Exchange on London in the countries named, and at London on the same countries, is quoted as follows:

United States, by giving the value of £1 in dollars and cents.
France and Belgium, by giving the value of £1 in francs and centimes.
Germany, by giving the value of £1 in marks and pfenninges.
Austria, by giving the value of £1 in florins and kreutzers.
Netherlands, by giving the value of £1 in guilders and cents.
India, by giving the value of 1 rupee in shillings and pence.

EXAMPLES.

431. *1.* Find the cost of a bill of exchange on London for £225 at 4.81¾.

ANALYSIS.—If £1 costs \$4.81¾, £225 will cost 225 times \$4.81¾.

2. What is the value of a draft for £324 16*s.* at 4.87½?

NOTE.—Write one-half of the greatest even number of shillings as tenths of a pound, and if there be an odd shilling write 5 hundredths. Thus, £324 16*s.* = £324.8; £324 17*s.* = £324.85. (See Art. **204**, Ex. 7, Note.) The value of £324 16*s.* at 4.87½ is found by multiplying \$4.87½ by 324.8.

3. Find the value of a draft on London for £379 12*s.* 7*d.*, at 4.86⅜.

OPERATION.

379.6
4.86⅜
949
475
22776
182208
14
1846.420

ANALYSIS.—If each penny be regarded as 2 cents, the result will be sufficiently accurate. For 11*d.* the maximum number of pence in any example, and exchange at 4.91, the error would be only ½ cent. \$4.86⅜ × 379.6 = \$1846.28. \$1846.28 + \$0.14 = \$1846.42. To save one addition, add the 14 cents to the partial products as in the operation.

Find the value of

4. £500 at 4.81½.
5. £775 at 4.85¼.
6. £837 at 4.83¾.
7. £84 8*s.* at 4.85.
8. £512 13*s.* at 4.84⅜.
9. £834 6*s.* 6*d.* at 4.88⅝.
10. £675 11*s.* 8*d.* at 4.87½.
11. £225 7*s.* 5*d.* at 4.82½.

12. Find the cost of a bill of exchange on Liverpool, for £875 12*s.* 6*d.* at the par value.

13. What are the proceeds of a draft of £959 5*s.* 4*d.*, sold through a broker, at 4.79½, brokerage ⅛%?

14. An exporter sold a draft for £540 3*s.* on Manchester, payable in London, at 4.84, brokerage ⅛%. What were the proceeds?

15. Find the proceeds of a draft on Newcastle-on-Tyne, at 60 days' sight for £1764 15*s.*, payable in London, at 4.82, brokerage on exchange ⅛%.

16. An importer purchased a bill of exchange on London, at 3 days' sight, for £488 16*s.* 6*d.*, at 4.85½. What was the cost?

17. How much exchange on London at 4.81¾ will $821.99 buy?

ANALYSIS.—$4.81¾ will buy exchange for £1; hence, $821.99 will buy as many pounds as $4.81¾ are contained in $821.99, or £170.625. £170.625 = £170 12*s.* 6*d.*

18. What will be the face of a 3 days' bill of exchange on London that can be bought for $5964.13, exchange 4.86½?

19. The face of a bill of exchange was £875, and its cost was $4233.91. What was the rate of exchange?

20. An exporter received $9063.22 for a bill of exchange that was sold through a banker at $4.86¾; what was the face of the bill, the broker's commission being ⅛%?

21. Find the cost of a bill of exchange on Paris for 7000 francs at 5.21⅞.

OPERATION.

5.21⅞) 7000
8 8
41.75) 56000.0000 (

ANALYSIS.—Since 5.21⅞ francs cost $1, 7000 francs will cost as many dollars as 5.21⅞ francs are contained times in 7000 francs.

Find the value of

22. 6000 francs at 5.16.
23. 5000 francs at 5.18⅛.
24. 4000 francs at 5.21⅜.
25. 8475 francs at 5.19½.
26. 7216 francs at 5.17¾.
27. 987.60 francs at 5.20⅞.

28. Find the cost of a draft on Antwerp at 3 days' sight, for 9640 francs, at 5.19⅜.

29. What is the value of a draft on London for £416 16*s.* 3*d.*, at 4.85⅜?

30. Bought exchange on Geneva, through a broker, for 8000 francs at 60 days' sight; what was the cost of the draft, exchange being 5.20⅝, brokerage ⅛%?

31. What is the cost of a draft on Paris for 12420 francs, at 5.19¾, brokerage on exchange ⅛%?

32. What will it cost to remit to Antwerp 8750 francs at the par value?

33. Sold through a broker a draft on Geneva for 7324 francs. What were the proceeds, exchange being 5.18⅜, brokerage ⅛%?

34. What will be the face of a bill of exchange on Geneva that can be bought for $15372, exchange selling at 5.22½?

35. Paid for a draft on Paris $3460.32; what was the face of the draft, exchange being 5.19⅜, and brokerage ⅛%?

36. A merchant paid \$6272 for a bill of exchange of 32512.48 francs; what was the rate of exchange?

37. Find the cost of a bill of exchange on Hamburg for 14400 marks (Reichsmarks) at 94⅛.

OPERATION.

4) 14400
3600
.94⅛
3388.50

ANALYSIS.—Since 4 marks cost \$0.94⅛, 14400 marks will cost 3600 (14400 ÷ 4) times \$0.94⅛, or \$3388.50.

Find the value of

38. 7200 marks at 94.
39. 8416 marks at 93½.
40. 3456 marks at 95¼.
41. 1237 marks at 93⅝.
42. 9894 marks at 95⅜.
43. 6515 marks at 94¾.

44. What is the cost of a bill of exchange on Frankfort for 16200 marks at 95½?

45. Sold a bill of exchange on Hamburg for 13200 marks, at 94⅛; what was the amount received, brokerage ⅛%?

46. An importer purchased a bill of exchange on London for £318 10*s.* 7*d.*, at 4.85¾; what did it cost?

47. What were the proceeds of a draft, sold through a broker, for 8748 marks, at 94⅜, brokerage ⅛%?

48. An exporter sold a draft on Paris for 12275 francs, at 5.19⅝; what were the proceeds, brokerage ⅛%?

49. What is the face of a bill on Hamburg that cost \$816, exchange 94⅛?

ANALYSIS.—Since \$.94⅛ will buy 4 marks, \$816 will buy 4 times as many marks as \$0.94⅛ is contained times in \$816.

50. What is the face of a 3 days' draft on Bremen, that was purchased in New York for \$3261.60, exchange 94⅜?

51. The cost of a draft of 12320 marks was \$2922.15; what was the rate of exchange?

52. Find the cost of a bill of exchange on Amsterdam, for 7240 guilders, at 40½.

53. Find the cost of a bill of exchange on Amsterdam, at 60 days' sight, for 12480 guilders, exchange 39⅞, brokerage ⅛%.

54. An exporter received \$1890.86 for a bill of exchange on Amsterdam; what was its face, exchange being 41⅛, brokerage ⅛%?

55. At 40⅜, how much exchange on Amsterdam will $2877.93 buy?

56. The value of a draft of 5280 guilders is $2145; what is the quotation?

57. The dividends of the N. Y. C. and H. R. R. Co., are paid in London at the rate of 49⅛ pence to the dollar. What is the equivalent rate of exchange?

58. Find the value in U. S. money of 16319 bushels of wheat at 4*s.* 4½*d.* per bushel, exchange 4.86½.

59. A merchant sent a messenger with a bill of exchange of 20000 francs to two bankers, A and B, with instructions to sell it to the best advantage. A offered 5.27 and B 5.27½. The messenger imprudently accepted the latter offer. How much did the merchant lose by the ignorance of the messenger?

60. When United States 4 per cent. consols are quoted in New York at 114½, and sterling exchange at 4.83½, what should be the London quotation of the bonds? What should be the London quotation of 4½ per cent. bonds, the New York quotation being 113¼?

NOTE.—In London, all American securities are quoted on an assumed value of the pound sterling of $5, instead of the actual value of $4.8665, or, more definitely speaking, its commercial value determined by the rate of exchange. Multiplying the New York quotation by 5 and dividing by the rate of exchange, the result will be the equivalent London quotation.

61. When American railway stocks are quoted in London at 88, what is the equivalent New York quotation, sterling exchange being quoted in New York at 4.88½?

62. What is the London equivalent of a New York quotation of 142, exchange being 4.83?

63. At Paris, what is the value of a draft on London of £550, exchange being 25.36½?

64. At London, what is the cost of a draft on Hamburg of 8000 marks, exchange being 20.45?

65. At Vienna, what is the cost of a draft on London of £625, exchange being 11.75?

66. At London, what is the value of a draft on Calcutta of 12000 rupees, exchange being quoted at 1*s.* 8$\frac{3}{16}$*d.*?

67. A commission merchant wishes to remit $2475 to his principal in England. How large a draft must he purchase, exchange being 4.83¼?

EQUATION OF ACCOUNTS.

DEFINITIONS.

432. Equation of Accounts (called also Equation of Payments and Averaging Accounts) is the process of finding the time when several debts due at different dates may be paid in one amount without loss of interest to either party. It is also the process of finding the time when the balance of an account having both debits and credits may be paid without loss of interest to either party. This time is called the *equated* or *average time.*

433. To find the equated time when the items of the account are all on the same side, i. e., all debits or all credits.

ANALYTICAL STEPS.—By assuming a certain date as the time of settlement, we find what the loss or gain of interest would be to the payer if all the bills were paid by him on that date. We next find in how many days the total amount of the bills would produce a sum equivalent to this loss or gain of interest, and find the true day of settlement by counting forward or backward this number of days from the assumed date. Thus, if the sum of the several bills is $1000, and the loss of interest to the payer at the assumed date of settlement is $10 (the interest of $1000 at 60 days at 6%), it is evident that the true date of settlement, or the time when there would be no loss of interest to either party, must be 60 days after the assumed date.

NOTES.—1. The interest on the bills paid after they became due would equal the interest on the bills paid in advance; the former being a gain to the payer, and the latter, a loss.

2. Any date may be assumed as the time of settlement. For convenience, the earliest or latest date is generally used. If the earliest date is taken, the estimated interest is a loss to the payer; if the latest is taken, the interest is a gain.

When the time is found by Compound Subtraction, or each month is regarded as 30 days, the last day of the month preceding the earliest item is the most convenient. (See second interest method.)

In Equation Tables, Dec. 31 or Jan. 1 is taken for all examples.

The assumed date is sometimes called the *focal date.*

3. Any rate of interest may be used in making the computations, 6 and 12 being the most convenient rates.

434. Ex. At what date may the following bills of merchandise be paid in one amount without loss of interest to either party? Due Apr. 10, $114; due Apr. 26, $140; due May 22, $320; due June 6, $976.

OPERATION.—PRODUCT METHOD.

Due Apr. 10,	$114	× 0 =	0	
" " 26,	140	× 16 =	2240	
" May 22,	320	× 42 =	13440	
" June 6,	976	× 57 =	55632	
	1550		) 71312	(46 days
				after Apr. 10, or May 26.

ANALYSIS.—For convenience, assume Apr. 10, the earliest due date, as the time of settlement. If the first bill, which is due Apr. 10, is paid on that date, there will be no loss or gain of interest to either party. If the second bill, which is due Apr. 26, is paid Apr. 10, 16 days before it is due, there will be a loss to the payer of the interest or the use of $140 for 16 days, or $2240 for 1 day. On the third bill, there will be a loss of the interest of $320 for 42 days, or $13440 for 1 day. On the fourth bill, there will be a loss of the interest of $976 for 57 days, or $55632 for 1 day. If all the bills are paid Apr. 10, there will be a loss to the payer of the interest of $71312 for 1 day, or of $1550 for 46 days. Since the loss of interest to the payer is equivalent to the interest of the total amount of the bills for 46 days, it is evident that the day when there would be no loss of interest must be 46 days after Apr. 10, or May 26. The payer is entitled to defer payment 46 days after the assumed date as a compensation for the estimated loss.

The gain of interest to the payer on the first three bills, which are paid after they are due, equals the loss of interest on the fourth bill, which is paid before it is due.

PROOF.

The interest of $114 for 46 days at 6% is	$0.874
" " " 140 " 30 " "	.70
" " " 320 " 4 " "	.213
Total gain of interest to the payer	1.787
The interest (a loss to the payer) of $976 for 11 days is .	1.789

NOTES.—1. In finding the number of days from the assumed date to the other dates, instead of calculating from the assumed date each time, find the interval from one date to the next and add it to the last number of days. Thus, from Apr. 10 to May 22 is 42 days, and from May 22 to June 6, 15 days; hence, from Apr. 10 to June 6 is 57 (42+15) days. (See Art. **210**, Ex. 3.)

2. To determine the due date, find the number of days in the operation nearest to the quotient, and add or subtract, as may be necessary, the difference between it and the quotient, to its corresponding date. Thus, in the above example, the number of days in the operation nearest to the quotient is 42; hence the due date is 4 (46-42) days after May 22, or May 26. (See Art. **254**, Ex. 3.)

3. If the fraction of the quotient is less than $\frac{1}{2}$, disregard it; if more than $\frac{1}{2}$, add 1 day to the integral number of days in the quotient.

435. RULE FOR THE PRODUCT METHOD.—*Assume the earliest due date as the day of settlement for all the items. Multiply each item by the number of days intervening between the assumed date of settlement and the date of the item; and divide the sum of the several products by the sum of the account. Count forward from the assumed date the number of days obtained in the quotient. The result will be the equated time.*

436. OPERATION.—FIRST INTEREST METHOD.

		Days.	Interest.
Due Apr. 10,	$114	0	$.00
" " 26,	140	16	.233 for 10 days. .14 " 6 "
" May 22,	320	42	1.60 " 30 " .64 " 12 "
" June 6,	976	57	4.88 " 30 " 2.44 " 15 " 1.952 " 12 "
	60) 15.50		) 11.885 (46 days
	.258		after Apr. 10, or May 26.

ANALYSIS.—Assume Apr. 10, the earliest due date, as the time of settlement. If the total amount ($1550) of the bills is paid Apr. 10, the assumed date of settlement, there will be a loss of interest to the payer of $11.885. The interest of $1550 for 60 days at 6% is $15.50, and for 1 day, $0.258. It will take $1550 to produce $11.885 interest as many days as $0.258 is contained times in $11.885, or 46 days. If, at the assumed date of settlement, there is a loss to the payer of the interest of $1550 for 46 days, the true day of settlement must be 46 days later, or May 26.

437. OPERATION.—SECOND INTEREST METHOD.

Mo.		Days.		Interest.			
0	Apr.	10,	$114	$0.19			
0	"	26,	140	.466	for	20	days.
				.14	"	6	"
1	May	22,	320	1.60	"	1	*mo.*
				1.067	"	20	days.
				.107	"	2	"
2	June	6,	976	9.76	"	2	*mo.*
				.976	"	6	days.
		2)	15.50				
			7.75	) 14.306 (1 *mo.* 25 *da.* after Mar. 31,			
				7.75		or May 25.	
				6.556			
				30			
			7.75)	196.680 (25 days.			
				1550			
				4168			
				3875			
				293			

ANALYSIS.—By this method, the last day of the month preceding the earliest due date is assumed as the date of settlement, and the time is found by Compound Subtraction, each month being regarded as 30 days.

The months are placed on the margin and the days correspond with the number of days in the given dates.

Mar. 31, the assumed day of settlement, there is a loss to the payer of $14.306 interest, or the interest of $1550 for 1 *mo.* 25 *da.* The equated time is therefore 1 *mo.* 25 *da.* after Mar. 31, or May 25.

Since this method regards all months as 30 days each, its results are not strictly accurate. The error in this example is 1 day. (See preceding results.)

When this method is used, and accurate results are required, the necessary corrections may be made by adding to the intervals of time 1 day for each intervening month containing 31 days. If the month of February is included, 2 days should be subtracted in a common year and 1 day in a leap year.

In counting forward to find the equated time, the opposite correction should be made. Thus, if the assumed date is June 30 and the quotient is 2 *mo.* 20 *da.*, the equated time would be Sept. 18, 2 days being subtracted for July and August.

The following is the corrected operation for the given example, 1 day being added to the time of the fourth item for the month of May. The result is the same as by the product and the first interest methods.

OPERATION.

Mo.	Days.		Interest.
0	Apr. 10,	$114	$0.19
0	" 26,	140	.466 for 20 days.
			.14 " 6 "
1	May 22,	320	1.60 " 1 *mo.*
			1.067 " 20 days.
			.107 " 2 "
2	June 6+1,	976	9.76 " 2 *mo.*
			.976 " 6 days.
	2)15.50		.162 " 1 "
	7.75		14.468 (1 *mo.* 26 *da.* after Mar. 31,
			7.75 or May 26.
			6.718
			30
			7.75)201.540 (26 days.

EXAMPLES.

438. *1.* At what date may the following bills be paid in one amount without loss of interest to either party? Due Sept. 10, $145; Sept. 28, $144; Oct. 8, $75; Oct. 23, $512.

2. What is the equated time for the payment of the following bills? Due Mar. 28, $446; May 3, $212; May 15, $116; May 31, $475; June 12, $345.

3. What is the average due date of the following bills, each being due at the date given? Jan. 5, $127.85; Jan. 26, $134.18; Feb. 5, $249.40; Feb. 23, $418.73; Feb. 28, $176.25.

NOTE.—The result will be practically the same if the nearest dollar is used in multiplying or in calculating the interest. Thus, in the above example, regard the amounts as 128, 134, 249, 419, and 176 respectively.

When there are several items in the example, some accountants omit the cents and units of dollars, and use the nearest number of tens. Thus, if the above account were of sufficient length, the numbers might be regarded as 13, 13, 25, 42, and 18 respectively. In this example the result is the same, but in some examples, containing the same number of items, there would be a discrepancy of one or more days.

4. Sold a customer bills at the due dates and to the amounts specified: June 1, $152.73; June 15, $114.28; July 16, $247.84; July 25, $88.90; Aug. 18, $735.42; Aug. 29, $416.34. When may the whole indebtedness be equitably discharged at one payment?

5. Average the following account:

NEW YORK, *July 1, 1882.*

MESSRS. RICE, STIX & Co.,

To LORD & TAYLOR, *Dr.*

1882.			
Apr. 4	Mdse. 30 days per bill rendered. . .	$816	37
" 21	" 30 " " " . .	724	25
May 13	" 30 " " " . .	342	46
" 25	" 30 " " " . .	535	84
June 16	" 30 " " " . .	628	62
	Due by equation June **.	****	**

NOTE.—When several bills are sold on a common term of credit, first find the average date of purchase, and to the result add the common term of credit.

Certain merchants sell uniformly on the same term of credit, while others sell on different credits, depending upon the class of goods, the standing of the customer, the state of the market, etc. (See Art. **275**.)

6. A. Hamilton bought of F. A. Leggett & Co., several bills of goods, as follows:

May 16, a bill of $212.46 on 60 days' credit.
" 28, " 318.40 " 60 " "
June 6, " 275.64 " 60 " "
" 21, " 187.83 " 60 " "
July 13, " 835.60 " 60 " "

A 60-day note for the whole amount is given in settlement. What must be its date, no allowance being made for the days of grace?

7. Sold on a credit of 90 days the following bills of goods: Mar. 4, $194.13; Mar. 27, $222.36; Apr. 12, $538.72; May 3, $432.64; May 28, $303.10. What is the equated time of payment? How much will settle the account Aug. 1, at 6%? How much July 1?

NOTE.—When monthly statements are sent to customers the accounts are frequently averaged. (See Ex. 5.) When the account is averaged, the simplest method of finding the cash balance due at a certain date, is to calculate the interest on the total amount from the average date to the time of payment, and add it, if the time of settlement is after the average date, and subtract it, if before.

Since a fraction of a day is not considered in determining the average date, this method of finding the cash balance is not as accurate as that of Art, , in which the interest is reckoned on each item separately.

8. A commission merchant sold several bills of goods, on a credit of 4 months, as follows: Aug. 16, 1881, $387; Sept. 4, 1881, $243.60; Sept. 18, 1881, $637.75; Oct. 28, 1881, $165.50; Dec. 10, 1881, $856.45. What is the equated time of payment?

NOTE.—The above account may be averaged by first finding the average date of purchase, and adding the common term of credit; or by finding the due date of each bill separately, and determining the average due date from the dates thus found. Since the months have not uniformly the same number of days, the results by the two methods sometimes differ by one or more days, when the common term of credit is expressed in months.

9. Bought goods on 6 months' credit as follows: Feb. 16, 1881, $376.50; Mar. 12, 1881, $287.40; Mar. 19, 1881, $612.87; Apr. 5, 1881, $345.60; Apr. 26, 1881, $134.80; June 1, 1881, $612.35. What is the average time of maturity? How much would balance the account Jan. 1, 1882? How much Oct. 1, 1881?

10. Park and Tilford sold to R. M. Bishop & Co. the following bills of merchandise on 60 days' credit: Feb. 24, $176.82; Feb. 28, $327.49; Mar. 16, $282.75; Mar. 28, $512.14; Apr. 7, $438.36; Apr. 14, $109.70; May 1, $632.65. What is the equated time of payment, and how much would be required to balance the account June 1? How much July 1?

11. The following bills of merchandise were purchased on 4 months' credit: June 1, $237.16; June 18, $146.75; June 30, $333.84; July 5, $416; July 16, $535.62; July 27, $912.33; Aug. 13, $345.60. A note payable in 4 months was given in settlement. What was its date, no allowance being made for the days of grace?

12. Bought goods on 60 days' credit as follows: Aug. 11, $487.60; Aug. 20, $398.30; Sept. 1, $411.26; Sept. 13, $283.36; Sept. 22, $112.43; Sept. 30, $555.55; Oct. 20, $342.48; Nov. 4, $337.64. What is the average due date?

13. What is the average time for the payment of the following bills, each being sold on a credit of 4 months? Feb. 29, $224.37; Mar. 13, $642.50; Mar. 31, $377.65; May 4, $510.10; May 19, $388.84; June 3, $476.25; June 19, $227.30; June 30, $562.75.

14. Bought several bills of goods as stated below:

June 3, a bill of $375 on 30 days' credit.
" 28, " 420 " 60 " "
July 16, " 560 " 4 months' "
Sept 4, " 228 " 90 days' "

What is the equated time of payment?

NOTE.—When the bills are sold on different terms of credit, first find the due date of each bill separately as in the following operation.

OPERATION.—PRODUCT METHOD.

Date of purchase.	Credit.	Due date.	Amount.	Days.	Products.
June 3,	30 days,	July 3,	$375 ×	0 =	0
" 28,	60 "	Aug. 27,	420 ×	55 =	*****
July 16,	4 *mo.*,	Nov. 16,	560 ×	*** =	*****
Sept. 4,	90 days,	Dec. 3,	228 ×	*** =	*****
			****		) ****** (** days.

OPERATION.—APPROXIMATE INTEREST METHOD.*

Mo.	Days.		Credit.	Interest.
0	June 3,	$375,	30 days,	$1.875 for 1 *mo.* .187 " 3 days.
0	" 28,	420,	60 "	4.20 " 2 *mo.* 1.68 " 24 days. .28 " 4 "
1	July 16,	560,	4 *mo.*,	11.20 " 4 *mo.* 2.80 " 1 " .933 " 10 days. .56 " 6 "
3	Sept. 4,	228,	90 days,	6.84 " 6 *mo.* .153 " 4 days.
		2) 15.83		
		7.915		7.915) 30.708 (3 *mo.* 26 *da.* after
				23.745 May 31, or Sept. 26.
				6.963
				30
				7.915) 208.890 (26 days.

* See second interest method, Art. 437.

15. What is the equated time for the payment of the following bills?

July	5, 1882,	$516.60	on	4 months'	credit.
	28, "	327.35	"	60 days'	"
Aug.	15, "	147.84	"	4 months'	"
Sept.	8, "	485.42	"	30 days'	"
"	25, "	230.39	"	60 "	"

16. Sold several bills of goods as follows:

May	4, a bill of	$418.75	on	30 days'	credit.
"	16, "	322.86	"	60 "	"
June	1, "	513.44	"	4 months'	"
"	12, "	118.70	"	60 days'	"
"	30, "	786.30	"	6 months'	"
July	16, "	274.85	"	60 days'	"

What is the average time of payment, and how much would balance the account Sept. 1? How much Oct. 1?

17. What is the average time of maturity for the payment of the following bills?

Mar.	4, 1883,	$117.26	on	4 months'	credit.
"	21, "	97.43	"	30 days'	"
"	29, "	243.84	"	60 "	"
Apr.	16, "	376.14	"	4 months'	"
"	30, "	182.75	"	90 days'	"
May	18, "	412.50	"	60 "	"
June	1, "	518.65	"	30 "	"

18. Bought goods of Henry Welsh as follows:

Nov.	13, 1881, a bill of	$138.42	on	30 days'	credit.
"	30, " "	416.10	"	60 "	"
Dec.	16, " "	324.70	"	30 "	"
Jan.	5, 1882, "	586.85	"	4 months'	"
"	26, " "	234.38	"	60 days'	"
Feb.	12, " "	93.60	"	4 months'	"
"	23, " "	618.75	"	30 days'	"
Mar.	5, " "	374.36	"	60 "	"

What is the equated time for the payment of the whole?

19. A commission merchant made the following sales for a consignor:

May	10,	$175,	on a credit of	4	months, or	30	days less 5%.
"	18,	243,	"	4	"	30	" "
"	31,	364,	"	4	"	30	" "
June	18,	387,	"	4	"	30	" "
July	1,	216,	"	4	"	30	" "

What is the average due date?

NOTE.—Since each of the above bills was sold on two different terms of credit, the account may be averaged on two different bases producing different results. The average date of purchase is June 5. If the account is settled on the first term of credit, the total amount of the bills, $1385, will be due 4 months after June 5, or Oct. 5. If the account is settled on the second term of credit, there will be $1315.75 ($1385 less 5%) due 30 days after June 5, or July 5. Since money is always worth less than 20% (4×5%), the second method is in favor of the commission merchant. Probably most of his buyers settle their bills on the second terms, and thus take advantage of the discount.

20. Average the following account on both terms of credit:

Jan.	16,	$387.65	on	6	months' credit less 4%	30	days.
"	28,	117.42	"	6	" "	30	"
Mar.	1,	482.60	"	6	" "	30	"
"	13,	618.32	"	6	" "	30	"
Apr.	4,	291.50	"	6	" "	30	"
"	11,	433.75	"	6	" "	30	"
"	23,	877.42	"	6	" "	30	"

21. A commission merchant made the following sales: Aug. 1, 1881, $387.40; Aug. 10, 1881, $416.75; Sept. 5, 1881, $583.28; Sept. 20, 1881, $144.13; Oct. 3, 1881, $582.76; Oct. 24, 1881, $327.41. What is the net amount and the average due date if the goods were sold on the following time? "60 days, or 2% discount if paid in 10 days."

22. A commission merchant sold the bills mentioned below on the following terms: Net 60 days, or 1% discount in 30 days, or 2% discount in 10 days. Apr. 19, $327.85; May 1, $282.64; May 13, $117.49; June 18, $486.40; June 30, $380.36; July 10, $516.64; July 17, $222.27. What is the net amount and the due date on each term of credit?

23. Average the following sales made by a commission merchant for a consignor:

Mar.	18,	$428.32	on	4 months'	credit,	or 30 days less 5%.
"	31,	385.74	"	60 days'	"	" "
Apr.	5,	212.50	"	4 months'	"	" "
"	26,	678.34	"	30 days'	"	" "
May	10,	824.60	"	4 months'	"	" "

NOTE.—If the 1st, 3rd, and 5th items are settled on the basis of 4 months' credit, the operation would be as follows by the product method:

OPERATION.

Due July	18,	$428.32	× 53	=	22684
" May	30,	385.74	× 4	=	1544
" Aug.	5,	212.50	× 71	=	*****
" May	26,	678.34	× 0	=	0
" Sept.	10,	824.60	× 107	=	*****
		2529.50		)	****** (** days after May 26.

NOTE.—If the 1st, 3rd, and 5th items are settled on a credit of 30 days less 5%, the operation would be as follows:

OPERATION.

Due Apr.	17,	$428.32	× 0	=	0
" May	5,	212.50	× 18	=	3816
" June	9,	824.60	× 53	=	43725
		1465.42			47541
Less 5%		73.27			2377
		1392.15			45164
" May	30,	385.74	× 43	=	*****
" "	26,	678.34	× 39	=	*****
		2456.23		)	***** (** days after Apr. 17.

24. Find the average time for the payment of the following sales:

Mar.	16,	$874.42	on 30 days'	credit.	
"	31,	555.37	" 60 "	"	
Apr.	5,	677.30	" 60 "	"	
"	16,	426.76	" 30 "	"	
"	24,	388.65	" 4 months'	"	or 30 days less 5%.
May	3,	112.60	" 4 "	"	" 30 " " 5%.
"	10,	989.10	" 60 days'	"	

25. Average the following sales:

Sept.	4, 1881,	$187.16	on	6 months'	credit,	or 30 days less 4%.
"	16, "	332.40	"	30 days'	"	
"	24, "	512.75	"	6 months'	"	or 30 days less 4%.
Oct.	5, "	164.60	"	6 "	"	" 30 " " "
"	27, "	187.30	"	6 "	"	" 30 " " "
Nov.	5, "	436.75	"	60 days'	"	
"	16, "	126.	"	6 months'	"	or 30 days less 4%.

26. Average the following account:

Dec.	1, 1882,	$246.75	on	30 days'	credit.	
"	12, "	312.40	"	60 "	"	
"	26, "	819.46	"	4 months'	"	less 5% 30 days.
Jan.	2, 1883,	674.32	"	4 "	"	" 5% 30 days.
"	10, "	126.60	"	60 days'	"	
Feb.	4, "	434.50	"	4 months'	"	less 5% 30 days.

439. To find the equated time for the payment of the balance of an account having both debit and credit items.

440. Ex. At what date may the balance of the following account be paid without loss of interest to either party?

Dr. JOHN ROACH in account with GEO. H. STUART. *Cr.*

1882.					1882.			
June 6	Mdse. 30 *da.*	456	00		July 26	Cash.	400	00
" 20	" 60 *da.*	384	00		Aug. 10	"	375	00
July 5	" 3 *mo.*	216	00		" 10	Mdse. 60 *da.*	288	00
" 26	" 3 *mo.*	552	00					

441. OPERATION.—PRODUCT METHOD.

Due	*Dr.*		Due	*Cr.*	
July 6,	$456 × 0 =	0	July 26,	$400 × 20 =	8000
Aug. 19,	384 × 44 =	16896	Aug. 10,	375 × 35 =	13125
Oct. 5,	216 × 91 =	19656	Oct. 9,	288 × 95 =	27360
" 26,	552 × 112 =	61824		1063	48485
	1608	98376			
	1063	48485			
	545	) 49891 (92 days after July 6, or Oct. 6.			

ANALYSIS.—First find the due date of each item. For convenience, assume July 6, the earliest due date, as the day of settlement for all the items on each side of the account. (See Art. **433,** Note 2.) If the balance of the account is paid July 6, the assumed date of settlement, there would be a loss to the payer, on the debit side of the account, equivalent to the interest of $98376 for 1 day, and on the credit side, of $48485 for 1 day; or a net loss of $49891 for 1 day, or of $545 for 92 days. Since the loss of interest to the payer by settling the account July 6, is equivalent to the interest of the balance, or the amount paid, for 92 days, it is evident that the day when there would be no loss of interest must be 92 days after July 6, 1882, or Oct. 6, 1882.

If the greater sum of the products had been on the credit side, there would have been a gain to the payer by settling the account July 6, and the day that the balance of the account would commence to draw interest would have been 92 days before July 6, or Apr. 5, 1882.

442. RULE FOR THE PRODUCT METHOD.—*First find the due date of each item. Assume the earliest due date as the day of settlement for all the items on both sides of the account. Multiply each item by the number of days intervening between the assumed date of settlement and the due date of the item, and find the sum of the products on each side of the account. Divide the balance (the difference between the sums of the debit and credit products) of the products by the balance of the account. The quotient will be the number of days intervening between the assumed date and the true date of settlement.*

To find the true date of settlement, count forward from the assumed date, when the balance of the account and the balance of the products are on the same side (both debits or both credits); and count backward, when on opposite sides.

NOTE.—1. The rule for counting backward and forward is the reverse of the above, when the latest date or a date after the latest date is taken as the assumed date of settlement.

2. Although the principles of equation of accounts are theoretically correct, they are not always practicable and can not be legally enforced. Thus, if a debt of $4000 is due Feb. 1, no merchant would accept a payment of $3600, Jan. 1, with the understanding that the remaining $400 would remain unsettled 9 months after Feb. 1, or until Nov. 1. The merchant would undoubtedly be willing to allow a discount equivalent to the interest of $3600 for the unexpired time, or 1 month.

3. In finding the equated time, reject the cents when less than 50; and add 1 dollar to the dollars when the cents are more than 50. The results will be sufficiently accurate.

443. OPERATION.—FIRST INTEREST METHOD.*

	Dr.				*Cr.*		
Due		Days.	Interest.	Due		Days.	Interest.
July 6,	$456	0	$0.00	July 26,	$400	20	$1.333
Aug. 19,	384	44	2.816	Aug. 10,	375	35	2.187
Oct. 5,	216	91	3.276	Oct. 9,	288	95	4.56
" 26,	552	112	10.304		1063		8.080
	1608		16.396				
	1063		8.08				
	60) 5.45		) 8.3160 (92 days after July 6, or				
	.0908						Oct. 6, 1882.

ANALYSIS.—If the account is settled July 6, the assumed date of settlement, Mr. R. would be entitled to a discount on the debit side of $16.396, and Mr. S. on the credit side of $8.08; or, Mr. R. would be entitled to a net discount of $8.316. If, by paying the balance of the account, July 6, Mr. R. is entitled to a discount of $8.316, it is evident that he should be allowed to defer payment until the balance would produce an equivalent interest, or 92 days. Hence, the true date of settlement is 92 days after July 6, 1882, or Oct. 6, 1882.

When the balance of the account and the balance of interest are both due the same party, the equated time is previous to the assumed date of settlement; and, when the balance of the account and the balance of interest are due different parties, the equated time is after the assumed date.

444. In the following operation, the latest due date is assumed as the date of settlement for all the items:

OPERATION.

Due		Days.	Interest.	Due		Days.	Interest.
July 6,	$456	112	$8.512	July 26,	$400	92	$6.133
Aug. 19,	384	68	4.352	Aug. 10,	375	77	4.812
Oct. 5,	216	21	.756	Oct. 9,	288	17	.816
" 26,	552	0	.00		1063		11.761
	1608		13.620				
	1063		11.761				
	60) 5.45	.0908	) 1.8590 (20 days before Oct. 26, or				
	.0908						Oct. 6, 1882.

ANALYSIS.—If the account is settled Oct. 26, the assumed date of settlement, the payer will be obliged to pay $1.859 interest in addition to the balance of the account. Hence, the date when the balance only may be paid without loss to either party must be 20 days before Oct. 26, 1882, or Oct. 6, 1882.

* See Art. 436.

445. OPERATION.—APPROXIMATE INTEREST METHOD.*

Dr.						*Cr.*			
Mo.	Days.	Credit.		Interest.	Mo.	Days.	Credit.		Interest.
0 June	6,	$456	30 *da.*	$2.28 .456	1 July	26,	$400		$2.00 1.333 .40
0 "	20,	384	60 *da.*	3.84 1.28					
1 July	5,	216	3 *mo.*	4.32 .18	2 Aug.	10,	375		3.75 .625
1 "	26,	552	3 *mo.*	11.04 1.84 .552	2 "	10,	288	60 *da.*	5.76 .48
		1608		25.788			1063		14.348
		1063		14.348					
		2) 5.45		2.725) 11.440	(4 *mo.* 6 *da.* after May 31, or				
		2.725		10.900					Oct. 6.
				.540					
				30					
				2.725) 16.200	(6 days.				

EXAMPLES.

446. *1.* At what date may the balance of the following account be paid without loss to either party?

Dr. ISAIAH B. PRICE. *Cr.*

1882.				1882.			
May 16	To Mdse.	437	00	May 23	By Cash.	400	00
" 31	" "	324	00	June 16	" "	300	00

2. Find the average date of maturity for the balance of the following account:

Dr. WILLIAM C. DOUGLAS. *Cr.*

1881.				1881.			
Jan. 4	Mdse. 30 *da.*	516	00	Feb. 1	Cash. . .	500	00
" 28	" 60 *da.*	325	00	" 1	Note 60 *da.*	300	
Feb. 4	" 4 *mo.*	437	00				

* See second interest method, Art. 437, and second method, Ex. 14, page 194.

3. Average the following account:

Dr. JOSEPH H. WRIGHT. *Cr.*

1882.					1882.			
Mar. 27	Mdse, 4 *mo.*	716	48		Apr. 16	Cash. . .	300	00
Apr. 16	" 60 *da.*	325	75		May 2	" . .	400	
May 1	" 4 *mo.*	413	40		July 8	" . .	500	
June 4	" 4 *mo.*	716	87					

4. What is the equated time for the payment of the balance of the following account?

Dr. A in account with B. *Cr.*

1882.				1882.			
Mar. 16	Mdse. 4 *mo.*	444	57	July 1	Cash. . .	400	
" 30	" 60 *da.*	376	82	" 20	" . .	375	
Apr. 20	" 30 *da.*	712	19	Aug. 16	" . .	700	
May 17	" 4 *mo.*	628	75	" 30	" . .	600	
" 28	" 4 *mo.*	419	31				

5. Average the following account. What will be the amount due Jan. 1, 1882?

Dr. C in account with D. *Cr.*

1881.				1881.			
June 16	Mdse. 30 *da.*	517	25	June 16	Note 60 (63) *da.*	1000	
" 28	" 60 *da.*	487	50	July 30	Cash. . .	375	
July 5	" 4 *mo.*	816	75	Aug. 13	Mdse. 4 *mo.*	900	
" 21	" 6 *mo.*	924	30	Oct. 5	Cash. . .	500	
Aug. 12	" 4 *mo.*	317	65				

6. When will the balance of the following account commence drawing interest? How much would be due Mar. 1, 1883.

Dr. ANDREW CARNEGIE, Pittsburg, Pa. *Cr.*

1882.				1882.			
Sept. 4	Cash	100	00	Aug. 16	Mdse. 4 *mo.*	647	13
" 4	Note 4 *mo.*	900	00	" 29	" 4 *mo.*	322	85
Oct. 31	Cash	250		Sept. 4	" 4 *mo.*	412	90
Dec. 28	"	600		" 17	" 4 *mo.*	588	33
				" 17	" 30 *da.*	246	12
				Nov. 4	" 4 *mo.*	683	45

7. Find the equated time for the payment of the balance of the following account.

Dr. JAMES B. FARWELL, Chicago, Ill. *Cr.*

1881.						1881.			
Jan. 4	Mdse.	4 *mo.*	637	20	Mar. 16	Cash.	300	00
" 14	"	4 *mo.*	412	87	Apr. 20	"	400	00
" 14	"	60 *da.*	214	35	May 3	"	200	00
Mar. 16	"	4 *mo.*	298	60	" 3	Note 4 *mo.*	800	00
" 28	"	30 *da.*	973	25				

8. Average the following account:

Dr. ARNOLD, CONSTABLE, & Co. *Cr.*

1882.					1882.			
Apr. 4	Mdse.	4 *mo.*	426	32	Apr. 25	Cash.	375	00
" 20	"	Cash.	387	40	June 30	"	600	00
May 13	"	60 *da.*	622	39	July 31	Note 60 *da.*	600	
" 27	"	30 *da.*	584	75	Aug. 15	Cash.	500	
July 5	"	4 *mo.*	224	50	Oct. 31	"	400	
" 16	"	4 *mo.*	838	95				

447. To find the equated time for the payment of the net proceeds (282) of an account sales (283).

448. 1. The sales form the credit side of the account, and the charges and advances the debit side.

2. The charges for transportation, cartage, and other items paid by the commission merchant are considered due at the time of the payment of the same.

3. The commission and other after-charges of the commission merchant are considered due by some at the average *due* date of the sales; and by others, at the average date of the sales. Since the commission is so small compared with the gross sales, in many examples, it makes no difference which date the commission is considered due. Certain merchants enter the commission at the date the account sales is rendered, and, by so doing, produce a result sufficiently accurate.

4. Many commission merchants, when the consignments are not separated and numbered, enter the sales and commission only on the account sales (See Ex. 4, Art. **450**), and enter the advances

and the general charges in the account current (See Ex. 1, Art. **456**). Accounts sales, when the shipments are continuous, are rendered monthly to the manufacturers or consignors, and "sketches" weekly or whenever a sale is made.

5. With the exception of finding the date for the commission and other after-charges, the process of averaging an account sales is exactly the same as that of averaging an account containing both debit and credit items.

449. Ex. What is the equated time for the payment of the net proceeds of the following account sales?

NEW YORK, Dec. 1, 1881.

Account sales of Seed

For account of WILLIAM STEPHENS & Co.

By FRANKLIN EDSON & Co.

1881.								
Nov.	4	45 20/60 *bu.* Timothy Seed . 30 *da.*	1 15/60	79	53		
"	18	50 " Mammoth Cl. Seed 60 *da.*	9 00/60	450			
"	28	49 48/60 " Clover Seed . . Cash.	8 40/60	418	32	947	85
		CHARGES.					
Oct.	31	Transportation.		60	00		
Dec.	1	Commission 5% as Dec. 22, 1881. .		47	39	107	39
		Net proceeds due Dec. 26, 1881. . .				840	46

ANALYSIS.—The average *due* date of the sales is Dec. 22, 1881, which is taken as the due date for the commission.

The account sales to be averaged will now be as follows:

Dr.		*Cr.*	
Due Oct. 31, 1881,	$60.00	Due Dec. 4, 1881,	$79.53
" Dec. 22, "	47.39	" Jan. 17, 1882,	450.00
		" Nov. 28, 1881,	418.32

By averaging the above, we find the net proceeds, $840.46, are due Dec. 26, 1881.

If the commission is considered due Nov. 21, 1881, the average date of the sales, the net proceeds will be due Dec. 28, 1881.

NOTE.—If the same assumed date, or focal date, be taken in finding the average due date of the sales as in finding the average due date of the net proceeds, the operation of the former will form the debit side of the latter operation.

ANSWERS.

Art. 20.

1. $3^2, 5, 7, 11.$
2. 3, 7, 11, 13.
3. $2, 3^3, 7, 11.$
4. $2, 3^2, 5^2, 7.$
5. $3, 5^2, 7^2.$
6. 2, 3, 5, 7, 11.
7. $2^3, 3^2, 7, 13.$
8. 2, 7, 13, 43.
9. 2, 3, 7, 11, 13.
10. $2^3, 11, 61.$
11. $3^2, 5^2, 17.$
12. $2^2, 11^3.$
13. $2^{13}.$
14. $3^6.$
15. $5^5.$
16. $2^3, 3^2, 5^2.$
17. $3^2, 5, 43.$
18. $3^2, 5^2, 11.$
19. $2^2, 3^2, 5, 37.$
20. $2^4, 3^2, 17.$
21. $5^2, 11, 31.$
22. $2^4, 3^3, 23.$
23. $2^3, 3^2, 7, 19.$
24. $5^2, 7, 29.$

Art. 26.

9. 36. *13.* 81.
10. 48. *14.* 126.
11. 55. *15.* 125.
12. 72. *16.* 144.

Art. 30.

1. 29. *6.* 6.
2. 37. *7.* 23.
3. 21. *8.* 1.
4. 43. *9.* 252.
5. 47. *10.* 17.

Art. 37.

8. 84. *13.* 480.
9. 360. *14.* 480.
10. 504. *15.* 72.
16. 480. *23.* 660.
17. 720. *24.* 756.
18. 360. *25.* 180.
19. 288. *26.* 420.
20. 528. *27.* 1980.
21. 1260. *28.* 2520.
22. 240.

Art. 41.

1. 1. *4.* 6.
2. $14\frac{2}{5}$. *5.* $\frac{5}{56}$.
3. $4\frac{23}{28}$. *6.* $51\frac{1}{2}$.
7. $41\frac{3}{8}\frac{4}{6}\frac{7}{6}$.
8. $29\frac{43}{75}$.
9. 2.
10. 1008.
11. $56.
12. $0.51.
13. $9.37\frac{1}{2}$.
14. 150 bu.
15. 750 yd.
16. 133 lb.
17. $31.50.
18. 36 cows.
19. 243 bu.
20. 1290 bu.
21. $16.92.
22. $79.
23. $21.
24. 56 lbs.
25. 66.
26. 10.
27. 90.
28. $23\frac{4}{17}$.

Art. 42.

2. 3,204,084.
3. 1,042.916,880.
4. $4799.50.
5. 9,741,862.
6. $18487.
7. 251.
8. 433.
9. 4653.
10. 862.72.
11. 16098.
12. 8899.35.
13. 4745.
14. 64535.
15. 45009.
16. 27369.
17. 41976.
18. 1145760.
19. $2^2, 3^3, 7, 11, 13.$
20. 92250.
21. 456.
22. 1802.
24. $458.
25. 1440.
26. 3582.
27. 132 acres.
28. 2520.
29. 2006.
30. $4.50.
31. 6.
32. $480.
33. $126.28.
34. $4664.24.
35. $46.
36. A, $12283;
B, $12568;
C, $12371;
D, $12071;
E, $12440;
D, Lowest.
37. 339 head.
38. $9292.80.
40. 3000 lbs.
41. $1044.
42. $.358;
$9.542.
43. $203.22.
44. $4,848,924.22.
45. A, $124;
B, $125.
46. $7.75.

Art. 65.

5. $\frac{9}{16}$. *18.* $\frac{13}{30}$.
6. $\frac{17}{21}$. *19.* $\frac{8}{9}$.
7. $\frac{2}{5}$. *20.* $\frac{5}{16}$.
8. $\frac{1}{2}$. *21.* $\frac{11}{15}$.
9. $\frac{3}{5}$. *24.* $\frac{49}{80}$.
10. $\frac{11}{14}$. *25.* $\frac{9}{16}$.
11. $\frac{7}{8}$. *26.* $\frac{3}{8}$.
12. $\frac{1}{5}$. *27.* $\frac{71}{195}$.
13. $\frac{54}{55}$. *28.* $\frac{71}{195}$.
14. $\frac{11}{12}$. *29.* $\frac{7}{15}$.
15. $\frac{3}{10}$. *30.* $\frac{16}{27}$.
16. $\frac{54}{91}$. *31.* $\frac{7}{15}$.
17. $\frac{81}{128}$. *32.* $\frac{6}{6}$.

Art. 68.

4. $\frac{75}{105}$; $\frac{63}{105}$; $\frac{147}{105}$.
5. $\frac{154}{56}$; $\frac{35}{56}$; $\frac{28}{56}$.
6. $\frac{42}{96}$; $\frac{8}{96}$; $\frac{57}{96}$.
7. $\frac{245}{360}$; $\frac{320}{360}$; $\frac{336}{360}$.
8. $\frac{51}{72}$; $\frac{44}{72}$; $\frac{42}{72}$.
9. $\frac{48}{108}$; $\frac{90}{108}$; $\frac{153}{108}$.
10. $\frac{200}{360}$; $\frac{135}{360}$; $\frac{132}{360}$.

Art. 73.

3. $\frac{55}{60}$; $\frac{32}{60}$; $\frac{160}{60}$.
4. $\frac{42}{48}$; $\frac{104}{48}$; $\frac{27}{48}$.
5. $\frac{50}{84}$; $\frac{70}{84}$; $\frac{84}{84}$.
6. $\frac{48}{80}$; $\frac{80}{80}$; $\frac{53}{80}$.
7. $\frac{20}{25}$; $\frac{15}{25}$; $\frac{21}{25}$.
8. $\frac{33}{72}$; $\frac{63}{72}$; $\frac{63}{72}$.
9. $\frac{102}{72}$; $\frac{117}{72}$; $\frac{80}{72}$.
10. $\frac{65}{100}$; $\frac{63}{100}$; $\frac{30}{100}$.
11. $\frac{117}{144}$; $\frac{144}{144}$; $\frac{84}{144}$.
12. $\frac{110}{225}$; $\frac{147}{225}$; $\frac{225}{225}$.

Art. 76.

1. $\frac{248}{8}$.
3. $\frac{208}{8}$.
4. $\frac{161}{4}$; $\frac{152}{8}$; $\frac{3337}{16}$.
5. $\frac{170}{8}$; $\frac{292}{8}$; $\frac{1872}{8}$.
6. $\frac{761}{2}$; $\frac{949}{10}$; $\frac{972}{5}$.

7. $\frac{179}{4}$; $\frac{449}{13}$; $\frac{817}{16}$.
8. $\frac{25}{8}$; $\frac{907}{20}$; $\frac{5445}{13}$.
9. $\frac{245}{8}$; $\frac{222}{8}$; $\frac{1503}{4}$.
10. $\frac{2211}{25}$; $\frac{5040}{11}$; $\frac{21455}{56}$.
11. $\frac{2889}{14}$; $\frac{2877}{16}$; $\frac{7071}{4}$.
12. $\frac{699}{8}$; $\frac{58885}{13}$; $\frac{20385}{18}$.
13. $\frac{2077}{8}$; $\frac{647}{4}$; $\frac{50225}{33}$.

Art. 79.

3. $93\frac{3}{4}$; 52.
4. 69; $125\frac{1}{8}$.
5. $57\frac{5}{9}$; $88\frac{1}{4}$.
6. $18\frac{15}{16}$; $35\frac{7}{12}$.
7. $36\frac{2}{15}$; $218\frac{1}{4}$.
8. $321\frac{3}{4}$; 78.
9. $49\frac{1}{9}$; $16\frac{17}{18}$.
10. $24\frac{3}{16}$; $13\frac{35}{52}$.
11. $20\frac{1}{24}$; $15\frac{37}{60}$.
12. $27\frac{2}{5}$; $38\frac{17}{18}$.
13. $24\frac{3}{16}$; $44\frac{1}{4}$.
14. $64\frac{5}{6}$; $48\frac{7}{8}$.

Art. 83.

1. $2\frac{1}{9}$.
2. $3\frac{1}{5}$.
3. $75\frac{3}{16}$.
4. $143\frac{2}{3}$.
5. $304\frac{5}{12}$.
6. $382\frac{47}{60}$.
7. $1105\frac{31}{90}$.
8. $378\frac{5}{16}$.
9. 278^{8}.
10. 566^{2}.
11. $140\frac{31}{72}$.
12. $270\frac{5}{28}$.
13. $1640\frac{1}{8}$.
14. $151\frac{131}{144}$.
15. $463\frac{1}{70}$.
16. $229\frac{11}{18}$.
17. $104\frac{19}{36}$.
18. 275^{2}.
19. 349^{1}.
20. $1586\frac{7}{16}$.

Art. 87.

1. $\frac{7}{88}$.
2. $\frac{11}{24}$.
3. $\frac{5}{24}$.
4. $\frac{15}{16}$.
5. $\frac{1}{6}$.
6. $\frac{4}{35}$.
7. $\frac{5}{16}$.
8. $\frac{19}{99}$.
11. $88\frac{3}{4}$.
12. $121\frac{5}{12}$.
13. $130\frac{1}{16}$.
14. $367\frac{1}{8}$.
15. $458\frac{7}{8}$.
16. $86\frac{1}{4}$.
17. $68\frac{1}{4}$.
18. $363\frac{11}{12}$.
19. $138\frac{1}{40}$.
20. $70\frac{1}{4}$.
21. $270\frac{5}{6}$.
22. $119\frac{1}{12}$.
23. $389\frac{1}{16}$.
24. $388\frac{15}{16}$.
25. $89\frac{11}{24}$.
26. $472\frac{7}{16}$.
27. 88^{2}.
28. 126^{3}.
29. 299^{1}.
30. 121^{2}.

Art. 91.

1. $\$171\frac{1}{4}$.
2. $922\frac{1}{2}$ bu.
3. $\$364$.
4. $\$368\frac{1}{2}$.
5. $\$21.22\frac{1}{2}$.
6. $8\frac{15}{16}$.
9. 1326.
10. 1884.
11. $745\frac{7}{8}$.
12. 440.
13. $1379\frac{7}{8}$.
14. $3364\frac{1}{2}$.
15. $4615\frac{1}{4}$.
16. 18024.
17. $19381\frac{5}{8}$.
18. 5955.
19. $36706\frac{1}{2}$.
20. $50383\frac{1}{8}$.
21. $2391\frac{1}{2}$.
22. $13136\frac{1}{4}$.
23. $10158\frac{3}{4}$.
24. 14865.
25. $77893\frac{3}{4}$.
26. $2048377\frac{1}{4}$.
27. $2520377\frac{1}{2}$.
28. 74571.
29. $289790\frac{3}{8}$.
30. 322887.
31. $3892291\frac{1}{2}$.
32. $79032\frac{1}{4}$.
33. $259546\frac{3}{8}$.
34. $183288\frac{1}{2}$.
35. $486354\frac{1}{8}$.
36. 340184.
37. $342468\frac{3}{4}$.
38. $413568\frac{1}{4}$.

Art. 95.

1. $\$2.18\frac{3}{4}$.
2. $\$11.14\frac{1}{4}$.
3. $\$36.65\frac{5}{8}$.
5. 72.
6. $30\frac{5}{8}$.
7. 1216.
8. 1287.
9. $2010\frac{1}{4}$.
10. 3105.
11. $9328\frac{1}{8}$.
12. 8500.
13. $4197\frac{3}{8}$.
14. $3091\frac{1}{4}$.
15. $4606\frac{1}{4}$.
16. 50570.
17. $19017\frac{1}{4}$.
18. $53462\frac{1}{2}$.
19. $57734\frac{5}{8}$.
20. $70977\frac{1}{4}$.
21. $47078\frac{1}{8}$.

Art. 98.

6. 16. 21. $\frac{10}{21}$.
7. 3. 22. $\frac{10}{13}$.
8. $55\frac{5}{6}$. 23. 20.
9. $\frac{1}{8}$. 24. $2\frac{17}{40}$.
10. $12\frac{1}{2}$. 25. 1152.
11. $11\frac{1}{21}$. 26. $2811\frac{1}{4}$.
12. $9\frac{1}{3}$. 27. 756.
14. $1\frac{71}{105}$. 28. 625.
15. $1\frac{9}{16}$. 29. $\frac{85}{144}$.
16. $3\frac{1}{2}$. 30. $1\frac{47}{144}$.
17. $13\frac{1}{2}$. 31. $\frac{1}{98}$.
18. $3\frac{1}{6}$. 32. $\frac{1}{7}$.
19. $\frac{32}{45}$. 33. $22\frac{1}{2}$.
20. $\frac{85}{272}$. 34. $\frac{25}{216}$.

Art. 102.

7. $57\frac{1}{2}$.
8. $145\frac{17}{76}$.
9. $116\frac{11}{52}$.
10. $81\frac{1}{15}$.
11. $70\frac{6}{7}$.
12. $72\frac{31}{32}$.
13. $214\frac{7}{12}$.
14. $576\frac{1}{2}$.
15. $483\frac{3}{4}$.
16. $703\frac{5}{7}$.
17. $809\frac{7}{18}$.
18. $1593\frac{37}{52}$.
19. $2143\frac{15}{16}$.
20. $410\frac{1}{4}$.
21. $1329\frac{1}{6}$.
22. $1428\frac{9}{10}$.
23. $576\frac{7}{12}$.
24. $194\frac{13}{16}$.
25. $1460\frac{7}{12}$.
26. $923\frac{7}{32}$.
27. $472\frac{1}{4}$.
28. $1019\frac{1}{4}$.
29. $810\frac{3}{5}$.
30. $396\frac{3}{5}$.

Art. 107.

3. $37\frac{1}{8}$. 23. 15.
4. 63. 24. 21.
5. $117\frac{1}{8}$. 25. 10.
8. $1\frac{1}{4}$. 26. $22\frac{1}{4}$.
9. $\frac{1}{12}$. 27. $12\frac{1}{2}$.
11. $6\frac{3}{4}$. 28. $16\frac{2}{3}$.
12. 9. 29. $18\frac{3}{4}$.
13. $11\frac{23}{37}$. 30. $21\frac{1}{2}$.
14. 9. 31. 15.
15. $6\frac{3}{12}$. 32. $9\frac{1}{2}$.
16. $2\frac{4}{15}$. 33. 13.
17. $2\frac{1}{7}$. 34. 46.
18. $1\frac{3}{7}$. 35. 13.
19. $\frac{9}{10}$. 36. 23.
20. $\frac{8}{15}$. 37. $33\frac{1}{3}$.
21. $26\frac{1}{4}$. 38. $13\frac{1}{2}$.
22. $7\frac{1}{2}$. 39. $13\frac{1}{3}$.
40. $\frac{9}{16}$; $\frac{9}{15}$; $\frac{11}{12}$.
41. $\frac{5}{64}$; $\frac{2}{3}$; $\frac{5}{6}$.
42. $\frac{3}{4}$; $\frac{6}{7}$; $1\frac{1}{2}$.
43. 2; $1\frac{1}{3}$.
44. $2\frac{1}{4}$; 2.
45. $635\frac{1}{3}$; $\frac{1}{3}$.

Art. 109.

4. $7441\frac{3}{4}$;
$32612\frac{3}{8}$;
$28419\frac{5}{8}$.
5. $7741\frac{1}{8}$;
$9031\frac{5}{16}$;
$32715\frac{15}{16}$.
6. $3845\frac{5}{8}$;
$14100\frac{5}{8}$;
$33200\frac{9}{16}$.
7. $7302\frac{11}{12}$;
$16233\frac{23}{24}$;
$20390\frac{15}{24}$.
8. $21765\frac{3}{8}$;
$68722\frac{5}{12}$;
$299475\frac{7}{16}$.
9. $39308\frac{23}{24}$;
$31386\frac{1}{12}$;
$35195\frac{1}{24}$.

Art. 110.

1. $\frac{9}{16}$.
3. $\frac{25}{8}$.
4. $17187\frac{1}{8}$.
5. $1931\frac{3}{24}$.
6. $862\frac{7}{12}$.
7. $3\frac{1}{2}$.
8. $3023561\frac{1}{2}$.
9. $3035201\frac{1}{2}$.
10. 10.

11. 12.
12. 21.
13. $1763\frac{3}{4}$; $352\frac{3}{4}$.
14. $186.00\frac{1}{4}$.
15. $145\frac{211}{240}$.
16. $2448.
17. $280.59.
18. $48492.
19. $95\frac{1}{2}$ bu.
20. $22.90\frac{1}{2}$.
21. $895\frac{1}{8}$.
22. A, $648; B, $1080.
23. $4360\frac{75}{128}$.
24. 336.
25. $2475.
26. $262\frac{1}{2}$.
27. $94\frac{1}{2}$.
28. $96\frac{1}{4}$; $36\frac{1}{2}$.
29. Horse, $705; Carriage, $440\frac{5}{8}$.
30. $2003\frac{1}{9}$.
31. $157.67.
32. $50\frac{1}{2}$.
33. $9.46.
34. $123\frac{1}{16}$ gal.
35. Widow, $2876.12; Each child, $1438.06.
36. $1403\frac{1}{2}$.
37. $192.
38. $4600.
39. Lost $.38$\frac{3}{20}$.
40. $20191\frac{3}{4}$; $24348\frac{7}{8}$.
41. $31963\frac{3}{8}$; $185517\frac{5}{32}$.
42. $6224\frac{5}{8}$; $14241\frac{3}{16}$.
43. $19479\frac{7}{8}$; $8426\frac{11}{16}$.
44. $198.31.
45. 19744.
46. $84.24.
47. 27.80.
48. 76.66.
49. 320 rods.
50. 6 days.
51. Gained 2 cts.
52. $629.30.
53. $5487.98.
54. 110 bu.
55. $35.46.
56. $136.99.
57. $115.30.
58. $316.74.
59. 319868; 26656.

Art. 129.

5. .4375.
6. .78125.
7. .425.
9. $.833\frac{1}{3}$.
10. $.583\frac{1}{3}$.
11. $.714285\frac{5}{7}$.
12. $.444\frac{4}{9}$.
13. 16.625.
14. $27.923076\frac{12}{13}$.
15. $36.9583\frac{1}{3}$.

Art. 132.

7. $\frac{2}{125}$. *19.* $\frac{5}{48}$.
8. $\frac{16}{125}$. *20.* $\frac{7}{72}$.
9. $\frac{9}{6250}$. *21.* $\frac{3}{560}$.
10. $\frac{64}{125}$. *23.* $\frac{1}{7}$.
11. $\frac{9}{16}$. *24.* $\frac{1}{12}$.
12. $\frac{3}{16}$. *25.* $28\frac{3}{80}$.
16. $\frac{5}{12}$. *26.* $107\frac{1}{6}$.
27. $175\frac{12}{125}$.
28. $6\frac{7}{400}$.

Art. 135.

1. 492.319787.
2. 7462.31526.
3. 476.3380807.
4. 2.6591587.
5. 9710.27879.
6. 1.83586255.
7. 1764.06.
8. $776.42\frac{7}{8}$.
9. $215.2742\frac{5}{12}$.
10. $21.9026730\frac{5}{9}$.

Art. 138.

1. 3.9803.
2. .26971.
3. 8999.1.
4. .4648.
5. 16.6736.
6. .010102.
7. $86.17.
8. 2.126155.
9. $1.728\frac{1}{4}$.
10. $121.14\frac{1}{2}$.
11. $1727.93\frac{1}{4}$.
12. $.924\frac{2}{3}$.
13. $31.0458\frac{1}{3}$.
14. $53.39\frac{1}{6}$.
15. $68.04\frac{11}{12}$.
16. $59.702\frac{8}{9}$.
17. $5512.33\frac{3}{7}$.
18. $931.057\frac{7}{9}$.
19. $.4657\frac{1}{6}$.

Art. 141.

1. .0004128.
2. .80448.
3. .00010112.
4. $43.216; f. 182.2875.
5. 45.77125; $55.02291\frac{2}{3}$.
6. .273735; 1.3136.
7. 3.39924; .409652.
8. .00540625; .8455375.
9. 11.208704; .0100672.
10. $5.705483\frac{1}{3}$; $34.01345\frac{5}{6}$.
11. $28.648083\frac{1}{3}$; 21.984375.
12. .288; 44.0928.
13. $93.0568\frac{1}{3}$; $4.0203\frac{1}{8}$.
14. $115.6666\frac{2}{3}$; $500.4029\frac{1}{6}$.
15. .51133; $3.8533\frac{1}{3}$.
16. $82.0166\frac{2}{3}$; $1061.1796\frac{7}{8}$.
17. 1296; 576; 432; 216; 345.6; 1080.
18. 25.8480726.
19. 170845.86.
20. .00132496; 430.5625; 2047.5625.
21. 520.688691125. .000669921875. 305841.443-359+.

Art. 144.

1. .048.
2. 250.
3. 104; 8.625.
4. 1.914; 2.82.
5. .875; 100.8.
6. 481.5; 385.2.
7. 4.25; 6.2.
8. 15.24706; 2.25.
9. .49; 82.6875.
10. .5694; 39.
11. 18.66; 10.30152+.
12. 2722.02; 42.
13. 86.40; 69.12; 51.84; 138.24 25.92.
14. 1800.
15. 3720.
16. 12.
17. 2.525.
18. 293.040015.
19. $3\frac{1}{3}$.
20. $158\frac{1}{3}$.

Art. 145.

1. .1651386.
2. $289.3624\frac{20}{21}$.
3. .8125.
4. $\frac{1}{12}$.
5. $158.916\frac{2}{3}$.
6. $176.277\frac{7}{9}$.
7. $\frac{67}{80}$.
8. 863.68964.
9. 3500.
10. .04.
11. 2.
12. $117.16\frac{2}{3}$.
13. 61.875.
14. 576.
15. 5.
16. $3\frac{1}{8}$.
17. 331.20.
18. $56.16.
19. $1575.
20. $108.99.
21. $143.06.
22. 21200 bu.
23. $5567.50; $4.443, wood; $.486, grain.
24. $1803.07.
25. $33107.12.
26. $10542.48, B: $7023.32, C, D, E.
27. $31444.87.
28. $203\frac{29}{80}$ Acres; $36\frac{39}{40}$ to B; $67\frac{63}{80}$ to C; $4575.65.
30. 7040 ft.
31. $500.
32. 275.7 tons; 118,754 tons; 430.7 persons.
33. 316+ tons; 92437 tons; 292 persons.
34. 3.07+ tons.
35. 48500, wife; 36375, son;

24250, daugh.; 145500, total.
36. 15½ hours.
37. 0.
38. Loss, $500.
39. $1065.
40. $4.63.
41. 1100.
42. 3423 1/14 miles.

Art. 196.

1. 2326 d.
2. 220 gills.
3. 108404 f.
4. 13265 lb.
5. 8784 hr.
6. 525600 m.
7. 235923 f.
8. 1566 qt.
9. 8760 hr.
10. 4480 pwt.
11. 27005 ft.
12. 2407680 ft.
14. 28675 l.
15. 2460 rd.
16. 7895½ sq. ft.
17. 2934¼ sq. yd.
18. 3200 a.
19. 16000 a.
20. 2160 cu. ft.
22. 35942 lb.
23. 474 qt.
24. 133000 lb.
25. $345.60.
26. 6574 d.
27. 266 d.

Art. 199.

1. £25 6s. 3d.
2. 75 C. 83 cu. ft.
3. 117 bu. 2 pk. 7 qt.
4. 38 bbl. 30 gal. 1 pt.
6. 252 sq. rd. 10 sq. yd.
7. 23 y. 42 d.
8. 37 wk. 6 d. 15 hr.
9. 2 hr. 38 m. 57s.
10. 31 rd. 2 yd. 2 ft. 5 in.
11. £55 15s. 7d.
12. 1704 f.
14. 247 ch. 36 ft.
15. 1346 ch.
16. 255 rd.
17. 33 A. 36 sq. rd.
18. .4938 A.
19. 6 m. 25 sq. ch.
20. 48 A. 5 p.
21. £18 8s. 2d.
22. $564.29.
23. $260.54.
24. $436.95.
25. $326.88.
26. 5177 1/2 ft.
27. $85.48.
28. $10.06.
30. £56 7s.

Art. 202.

1. 150 d.
2. 17s. 6d.
3. 135 d.
4. 11s. 3d.
5. 2 y. 4 m.
6. £16 9s. 4d.
7. £2053 7s. 9d.
8. 2 y. 5 m.
9. 36 cu. yd. 3 cu. ft.
10. 3 m. 3 d.

Art. 204.

1. £ 1/3 3 6 4.
2. £.04375.
4. 7/8 oz.
5. £.008125.
6. 1/5 2 8 0 0 mi.
7. £.925.
8. .90625 cd.
9. £247.7875.
11. 5/5 2 8 8 mi.
12. £27.525.
13. $42.65.
14. $166.56.
15. $138.01.
16. $146.56.
17. $6022.92.
18. £1 5s. 7½d.

Art. 206.

1. £85 7s. 10d.
2. 69°T. 15 cwt. 20 lb.
3. £460 7s. 6d.
4. 12 yr. 2 mo. 16 da.
5. 21 cd. 34 cu. ft.
6. 26 hr. 46 min.
7. 152° 4′ 2″.
8. 121.31 ch.
9. 10 lb. 2 pwt. 20 gr.
10. 124 gal. 2 qt.
11. £205 12s. 5d.
12. £546 1s. 7d.

Art. 208.

1. £88 4s. 2d.
2. £21 4s. 4d.
3. £3 14s. 3d.
4. £23 18s. 10d.
5. 4 yr. 3 mo. 2 da.
6. 4 yr. 4 mo. 22 da.
7. 6 mo. 10 da.
8. 2 yr. 6 mo. 24 da.
9. 1 hr. 58 min. 4 sec.
10. 17° 53′ 56″.
11. 17 T. 14 cwt. 24 lb.

Art. 211.

1. 7 mo. 18 da.
2. 8 mo. 18 da.
3. 1 yr. 8 mo. 11 da.
4. 2 yr. 5 mo. 7 da.
5. 1 yr. 10 mo. 8 da.
6. 9 mo. 12 da.
7. 9 mo. 12 da.
8. 1 yr. 4 mo. 15 da.
1. 230 days.
2. 263 days.
3. 619 days.
4. 888 days.
5. 678 days.
6. 286 days.
7. 286 days.
8. 504 days.

Art. 213.

1. £157 16s.; £192 17s. 4d.; £263.
2. 34 cd. 16 cu. ft.
3. £28 2s. 6d.
4. £6 7s. 10d. £7 5s. 9d.
5. 3 lb. 3 oz. 12 pwt.
6. 24 hr. 35 min. 30 sec.; 19 hr. 40 min. 24 sec.; 29 hr. 30 min. 36 sec.
7. 55 da. 13 hr.
8. £3 9s. 2d; £8 6s. 1d; £9 13s. 10d.
9. £20 12s. 6d.
10. 167 bu. 1 pk.
11. 44 mi. 6 fur.; 71 mi. 4 fur. 32 rd.

Art. 215.

1. £1 4s. 9d.; £1 10s. 3d.; £ 8s. 3d.
2. 192 yd.; 324 yd.
3. £17 5s.; £9 11s. 8d.; £5 15s.
4. 44 yd.
5. 5° 41′ 14″; 4° 44′ 21 3/5″; 3° 9′ 34 4/5″.
6. £41 18s. 11d.
7. 32 A. 41 sq. rd. 24 A. 130 sq.rd. 20 A. 25 5/8 sq.rd.
8. 99 ft.
9. 3 oz. 5 5/8 pwt.
10. 9s. 11d.; £5 19s.; £9 18s. 4d.

Art. 220.

1. 73° 54′ 25″.
2. 73° 23′ 52″.
3. 34° 49′.
4. 178° 34′ 17″.
5. 88° 33′ 45″.
6. 149° 14′ 13″.
8. 54 min. 30 4/5 sec.
9. 11 hr. 2 min. 59 8/15 sec.
10. 50 min. 11 7/15 sec.
11. 5 hr. 4 min. 20 4/5 sec.
12. 11 hr. 58 min. 11 13/15 sec.
14. 154° 8′ 30″ W.
15. 5 hr. 8 min. 42 sec., P. M.
16. 19° 31′.
17. 10 hr. 54 min. 27 sec., A. M.

Art. 226.

5. 8742.57 m.
6. 119 Km.

7. $98.70.
8. 2306.8 m.
9. 910 m.

Art. 229.

6. 400.
7. 166.4 H.
8. $12.44.
9. 40.
10. 96 H.
11. 650.

Art. 232.

3. 28 cu. m.
5. 4.81208 cu. m.
6. 2051.28 cu. m.
7. $109.69.
8. 82.5 sters; 43.2 m.

Art. 234.

3. 72 Hl.
4. 168 Hl.
6. $1498.
8. 180 bags.
9. 10080 Dl.
10. $14.

Art. 236.

2. 16.816 T.
3. 80.
4. 30.
5. 750 g.
6. $13.608.
7. 1200.
8. 62.
9. 9300 Kg.

Art. 239.

1. 246.06 yd.; 8858.25 in.
2. 9.6558 Km.
3. 259.008 H.
4. 32808.3 ft.; 6.2137 mi.
5. 828.04776 lb.
6. 204.12.
7. 26.73 grams; 27.216 grams.
8. 1762 Hl.
9. 668.9375 cu.
10. 1308 cu. yd.
11. 6540.48 l.
12. 291.824 sq. yd. 2626.416 sq. ft.
13. 6237 g.; 6.237 kg.

Art. 260.

1. 202; 178.
2. 1 y. 4 m. 22 d.
3. 4; 12; 4.
4. 2 y. 4 m. 15 d.
5. 69$\frac{6}{9}$¢.
7. 66 ch. 20 ft.; 52512 in.
8. 99 ch.; 6534 ft.
9. 2478.96 ft. 150.24 rd.
10. 6$\frac{1}{8}$ in.; 8$\frac{1}{8}$ in.; 11$\frac{1}{8}$ in.
11. 2999.875 sq. ft.
13. 132.183 A.
14. 1.00352 A.
16. 23$\frac{5}{9}$ sq. yd.
17. $68.27.
18. $7200.
19. 990 ft.
20. $72.
21. 8 ft.
22. 240 ft.
23. 240 ft.
24. 2700 ft.
25. $81.82.
26. 16500 ft.
27. $327.78.
28. 1400.
29. 76.8 oz. av.
30. 57.6 lb. av.
31. $141.87.
32. $84.31.
33. $172.33.
34. $127.49.
35. 487$\frac{11}{12}$ bu.; $237.81.
36. $237.94.
37. 229$\frac{11}{12}$ bu.
38. $158.54.
39. $4.92.
40. $5.09.
41. $6.98.
42. $7.20.
43. $0.687; $0.531; $1.02; $0.935.
44. 455$\frac{11}{12}$; 435$\frac{11}{12}$; 506$\frac{25}{36}$; 446$\frac{1}{4}$; 511$\frac{11}{12}$.
45. 90; 155$\frac{20}{21}$; 143$\frac{24}{25}$; 110$\frac{20}{21}$; 197$\frac{2}{3}$.
46. $3.091; $1.362; $1.523; $1.812.
47. $0.762; $0.741.
48. $3.889; $14.25.
49. $430.72.
50. $1634.09.
51. $808.65.
53. $0.351.
54. 162145.26 g., gold; 18016.14 g., alloy.
55. 1546875 oz., silver; 171875 oz., copper.
56. 8355 lb. 7 oz. 98.9 g.
57. $18.605; $16.957.
58. $20.672; $18.842.
59. $0.8392.
60. 40,608,021 pieces; $88,312,142.50.
61. £135 1s. 10d.
62. £49 7s.
63. £219375; $23.72; $1067588.44.
64. 40 yd.
65. £19.8375.
66. $1100.25.
67. £15 16s. 1d.
68. £10 10s.
69. 5.1813 f.
70. $3377.50.
71. 412.50 f.
72. $5712.
73. $3018.49; $7977.96.
74. 43 r. 9 a. 11 p.
75. $246.75.
76. .9 fine; $0.9969; 6.
77. 55° 48′.
78. 4 h. 23 m. 47$\frac{2}{3}$ s., P. M.
79. 160.71 bu., exact; 160 bu. app.
80. 320 Hl.
81. 4189$\frac{1}{11}$ gal.
82. 165000 l.
83. 56.867 yd.
84. $0.526.
85. 46.45 sq. dm.
86. $6.889.
87. $0.528.
88. $0.055.

Art. 273.

5. $1.25.
6. 60.
7. 1.94.
8. 21.90.
9. $125.85.
10. 159.56\frac{1}{4}$.
11. $28.75.
12. $112.68.
13. $2731.25, paid over.
14. $225.
15. $832.58.
16. 2500.
17. 2400.
18. $361.60.
19. $10800.
20. $1502.40.
21. $72.
22. 495 ft.
23. £7200.
24. $200.
25. 12000.
26. $15627.50.
27. $91500.
28. $2.94.
29. 2200.
30. $65500.
31. $1612.50, A; $2700.40, B.
32. .01$\frac{1}{4}$.
33. 33$\frac{1}{3}$%.
36. 2$\frac{1}{12}$%.
37. 3$\frac{1}{3}$%.
39. 6$\frac{1}{4}$%.
40. 2$\frac{1}{2}$%.
41. 1$\frac{3}{4}$%.
42. 25%.
43. $\frac{3}{20}$%.
44. 73.57%; 26.43%.
45. 12$\frac{1}{2}$%.
46. 2325.37.
47. $564.
49. $930.
50. $444.
51. $324.
52. 6210.
53. $456.80.
54. $2555.91.
55. $1392.
58. $1197.
60. $7500.
61. 22.99%.
62. 48.39%.
63. 76.19%.
64. 20.14%.
65. 23.61%.
66. £18 15s.
67. £1 12s. 5d.

68. £3 0s. 6d.
69. £3 14s. 10½d.
70. £32.
71. £35 8s. 4d.

Art. 276.

1. $78.55.
2. $31.50.
3. $206.55.
4. $76.58.
5. $40.30.
6. $18.24.
7. 23⅓%; 50½%; 30%; 64%; 78¼%.
8. $102.82.
9. $420.26.
10. $535.43.
11. $147.67.
12. $40.83.
13. 200 yd.
14. $354.06.
15. $26.25.
16. $168.93.
17. $905.23; $896.08.
18. $450.95.

Art. 278.

1. $44.10.
2. $926.12.
3. $848.27.
4. $533.22.
5. $135.73.
6. $118.88.
7. $360.07.
8. $900.43.
9. $150.30.
10. $341.64.
11. $1082.55.
12. $145.
13. $65.64.
14. $70.21.
15. $266.04.
16. $94.81.

Art. 286.

1. $21.60.
2. $200000.
3. $212.50.
4. $2256.25.
5. $2295.
6. 2%.
7. 480
8. $3099.37.
9. $12.61.
10. $81.75.
11. $2161.17, net amount.
12. $65.63.
13. $869.60, com.
14. $125.
15. 5%.
16. $5091.
17. $2545.75.
18. $43500.
19. $1773.50.
20. $8450.89.
21. $2782.30.
22. $7872.07.
23. $2175.44.
24. $50000.
25. $18909.18.
26. 135458 lb.
27. $2751.14.
28. $123.31.

Art. 289.

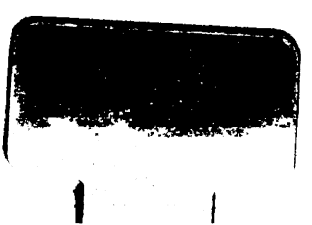
1. $40.
2. $364.
5. $4726.
7. $3.
8. $10179.
10. $1048.20.
11. 72¢.
12. $2425.
14. 16%.
15. 13$\frac{23}{25}$%.
16. 12½%.
17. $1775.
18. $608.
19. $11250.
20. $520.
21. 44¢.
22. $72.
23. 23⅛%.
24. 150%.
25. 57½¢.
26. 12%.
27. 10%.
28. 4%.
29. $210.
30. $50.
31. $7.50.
32. 36¢.
33. 42$\frac{6}{7}$%.
34. 40$\frac{5}{8}$%.
36. 33⅓%; 50%; 53$\frac{11}{18}$%; 40$\frac{20}{27}$%; 42$\frac{6}{7}$%; 80$\frac{10}{18}$%.
37. 6¾%, gain; $3.80.
38. $2532.96; $3102.88; 22⅛%.
39. $400.
40. 9$\frac{1}{11}$%.
41. $2.50.

Art. 303.

1. $122.50.
2. $115.65.
3. $525.25.
4. $702.51.
5. $917.72.
6. $214.31.
7. $535.50.
8. $397.49.
9. $106.58.
10. $3393.33.
11. $1059.71.
12. $470.63.
13. $83.55.
14. $18.81.
15. $1791.94.
16. $269.45.

Art. 314.

1. $16.28.
2. $64.76.
3. $36.85; $42.99.
4. $24.70; $20.58.
5. $56.68; $75.57.
6. $180.10; $120.07.
7. $11.43; $17.14.
8. $15.50; $18.08.
9. $29; $33.83.
10. $33.17; $44.23.
11. $9.19; $7.66.
12. $16.38; $10.92.
13. $39.45; $46.03.
14. $19.79; $23.09.
15. $2.85; $2.37.
16. $13.75; $16.04.
17. $106.66; $142 22.
18. $137.72; $114.77.
19. $107.36; $80.52.
20. $93.53; $140.30.
21. $19.27; $14.46.
22. $494.15; $247.07.
23. $418.64; $837.29.
24. $17.85; $20.83.
25. $158.29; $211.05.
26. $48.25; $40.21.
27. $53.51; $35.67.
28. $188.40; $282.60.
29. $104.93; $78.69.
30. $5.64; $2.82.
31. $154.87; $309.74.
32. $717.50; $657.71.
33. $34.96; $33.21.
33. $35.65; $33.87.
34. $14.27; $18.73.
34. $14.47; $18.99.
35. $26.73; $19.09.
35. $27.29; $19.49.
36. $193.96; $113.14.
36. $195.15; $113.83.
37. $309.07; $347.70.
37. $310.14; $348.90.
38. $57.27; $81.81.
38. $57.82; $82.60.
39. $7.59; $10.12.
40. $25.95; $11.53.
41. $111; $27.75.
42. $76.50; $60.56.
42. $77.40; $61.28.
43. $16.28; $14.65.
43. $16.57; $14 92.
44. $10658.20.
45. $1050.
46. $1556.66.
47. $27.84.

Art. 318.

1. $1.79.
2. $116.47.
3. $11.54.
4. $5.29.
5. $3.92.
6. $15.12.
7. $5.75.
8. $6.16.
9. $42.18.
10. $14.83.
11. $4.44.
12. $10.44.
13. $39.35.
14. $246.89.
15. $58.97.
16. $27.74.
17. $41.64.
18. $7.58.
19. $2.14.
20. $30.21. 21. $5.